Nonlinear Nonlinear Model Predictive Control

Theory & Implementation

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CNRS / Gipsa-lab / Control Systems Department

(Journée CCT-CNES, Toulouse, 15 Janvier 2009)

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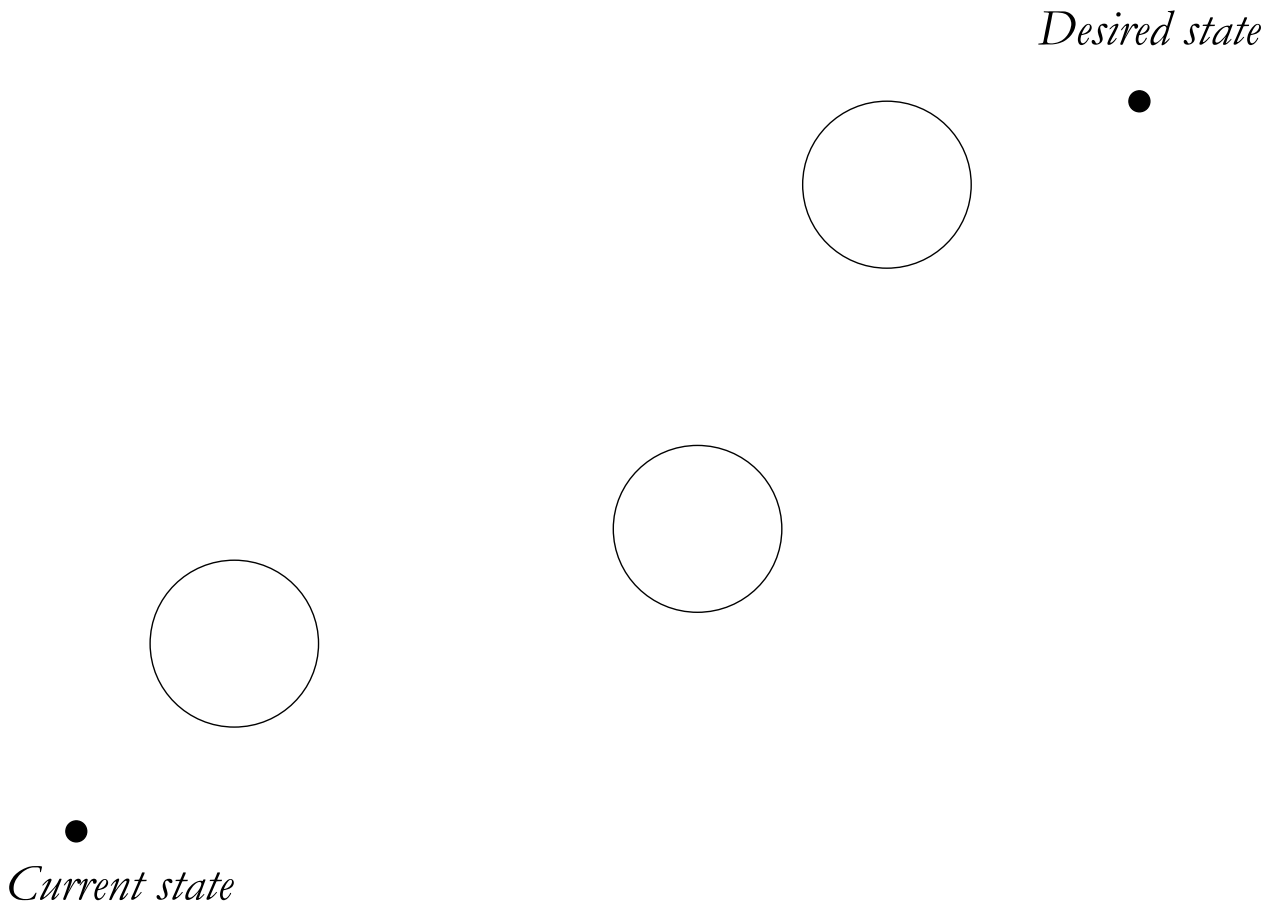

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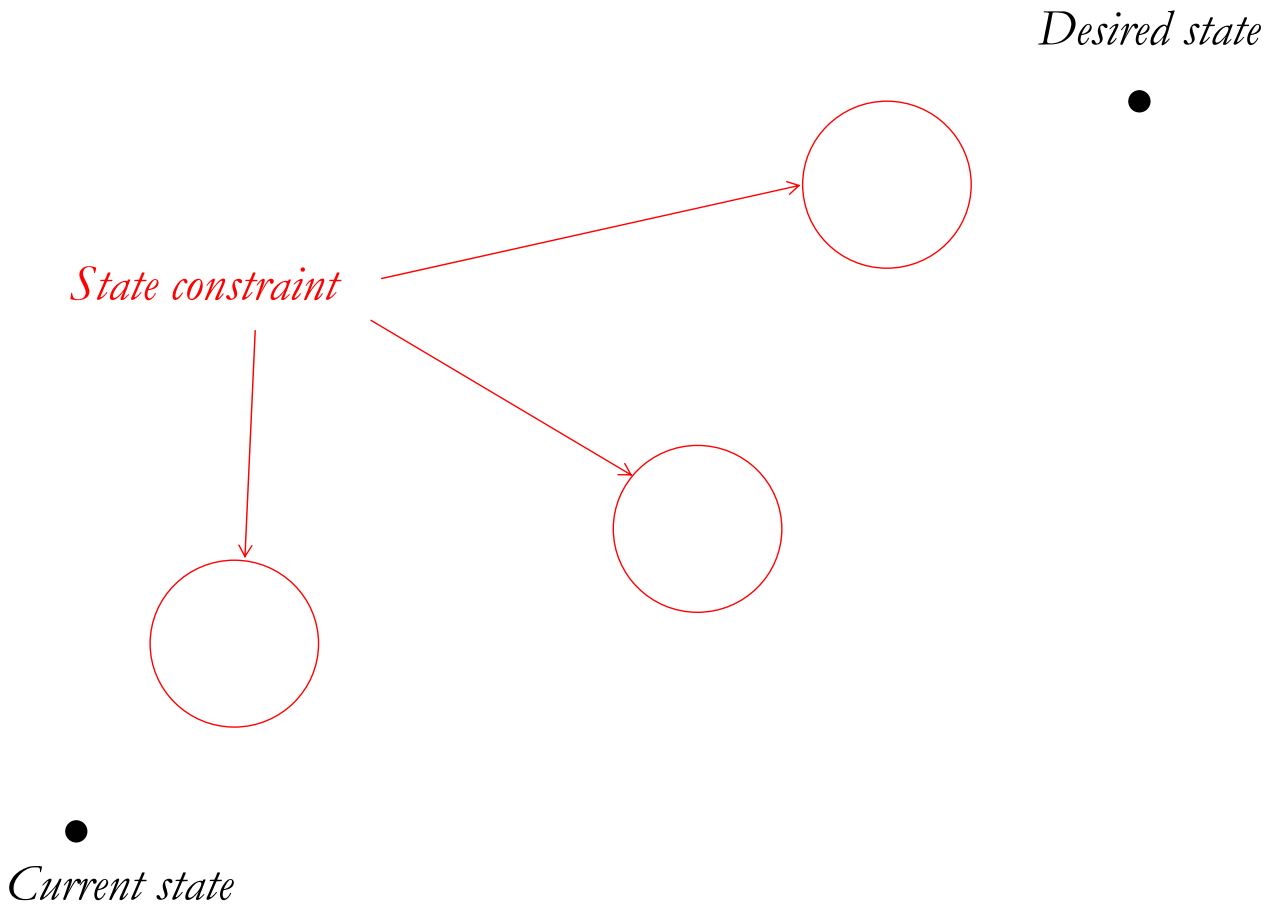

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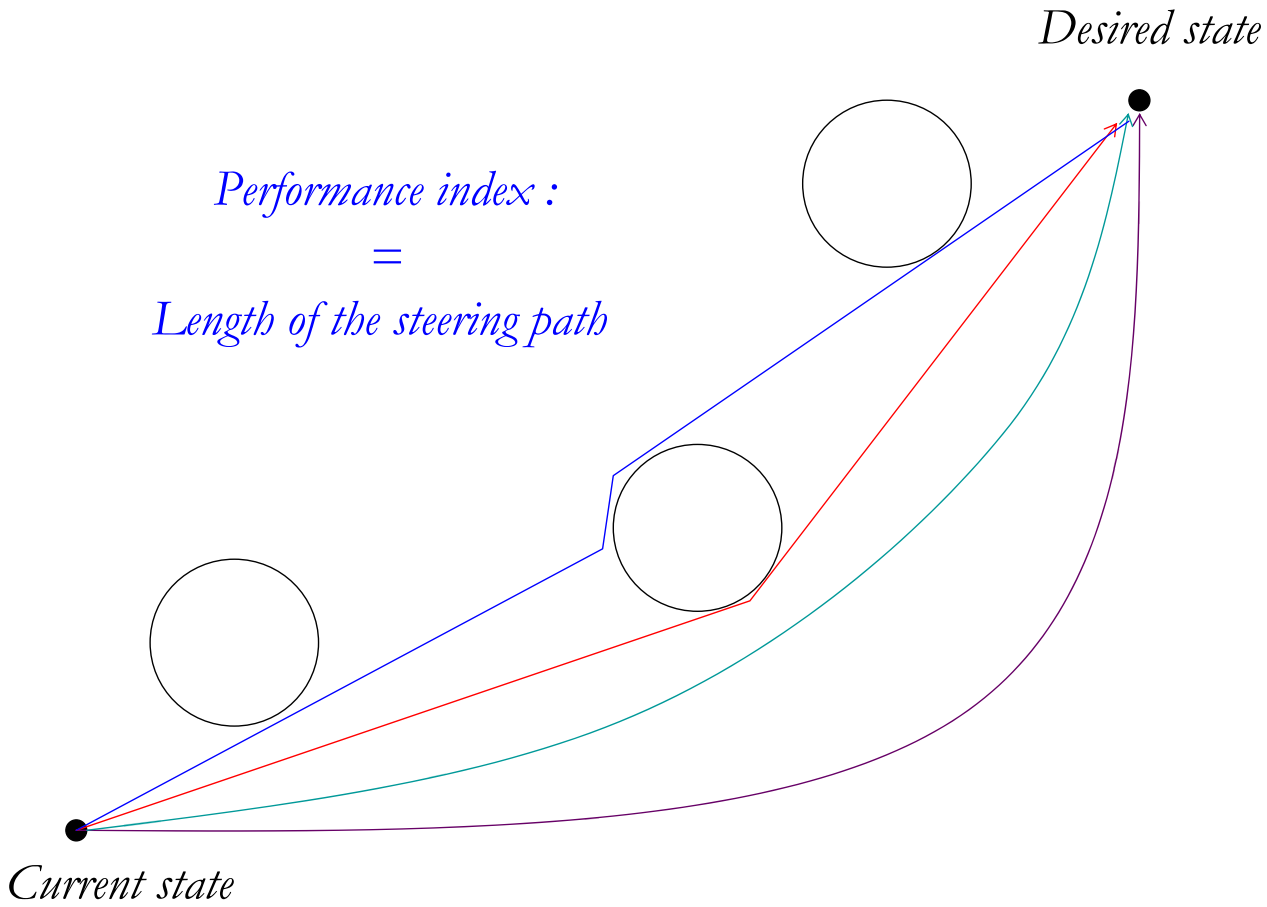

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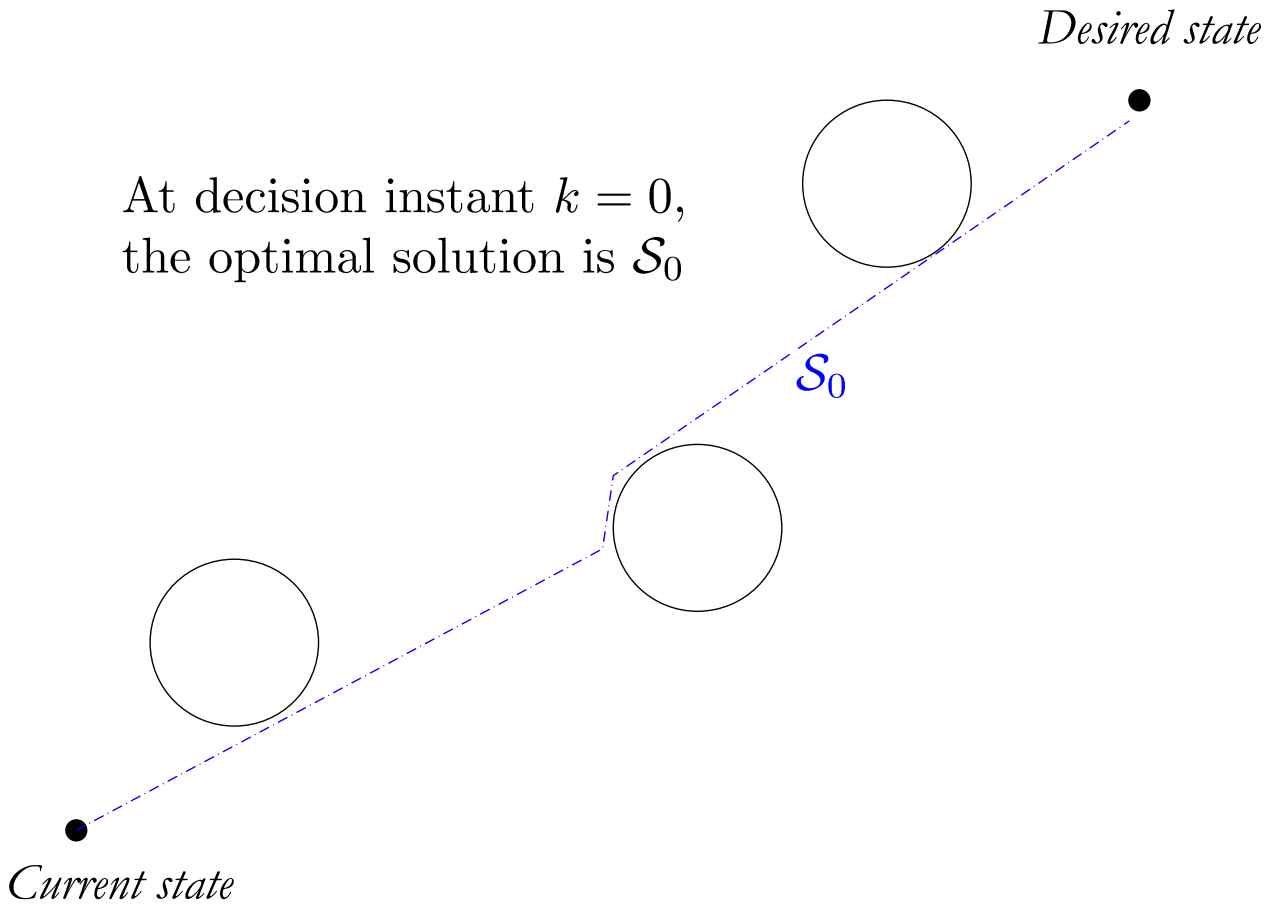

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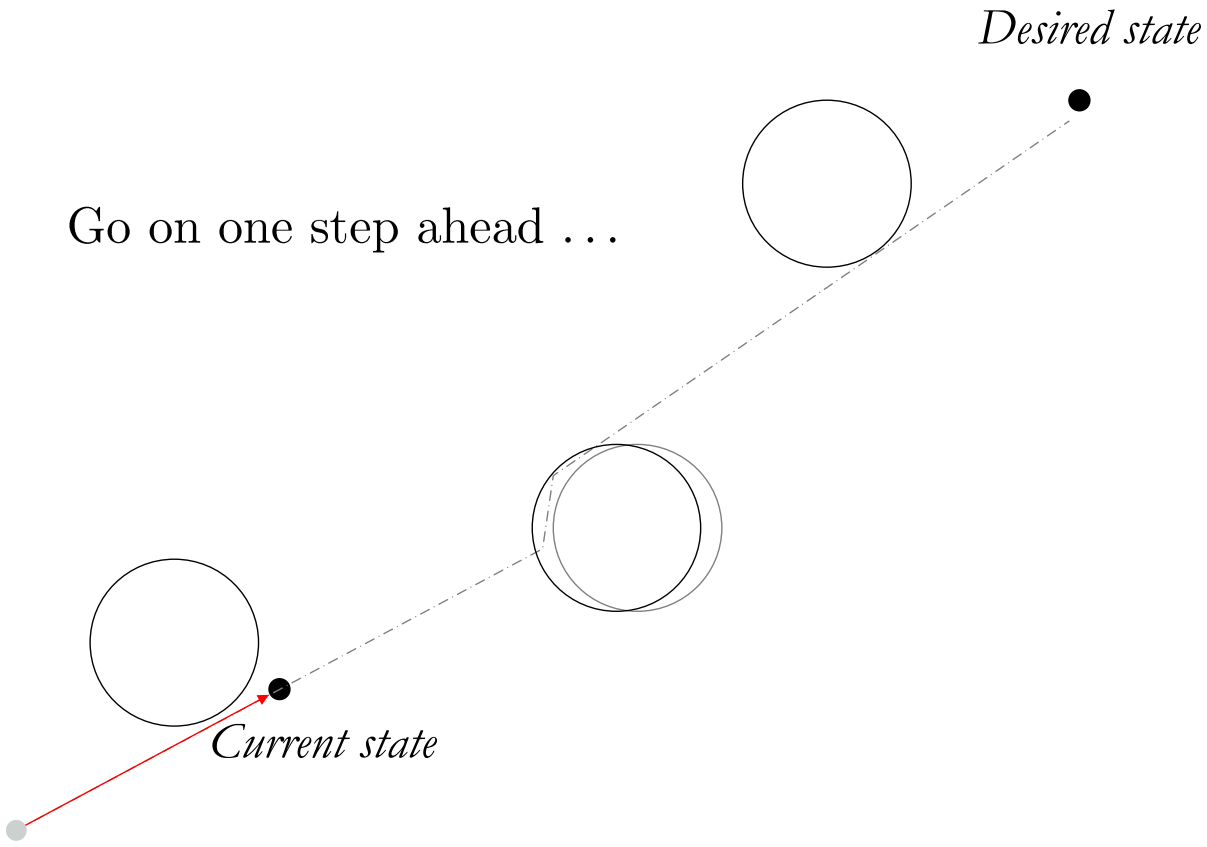

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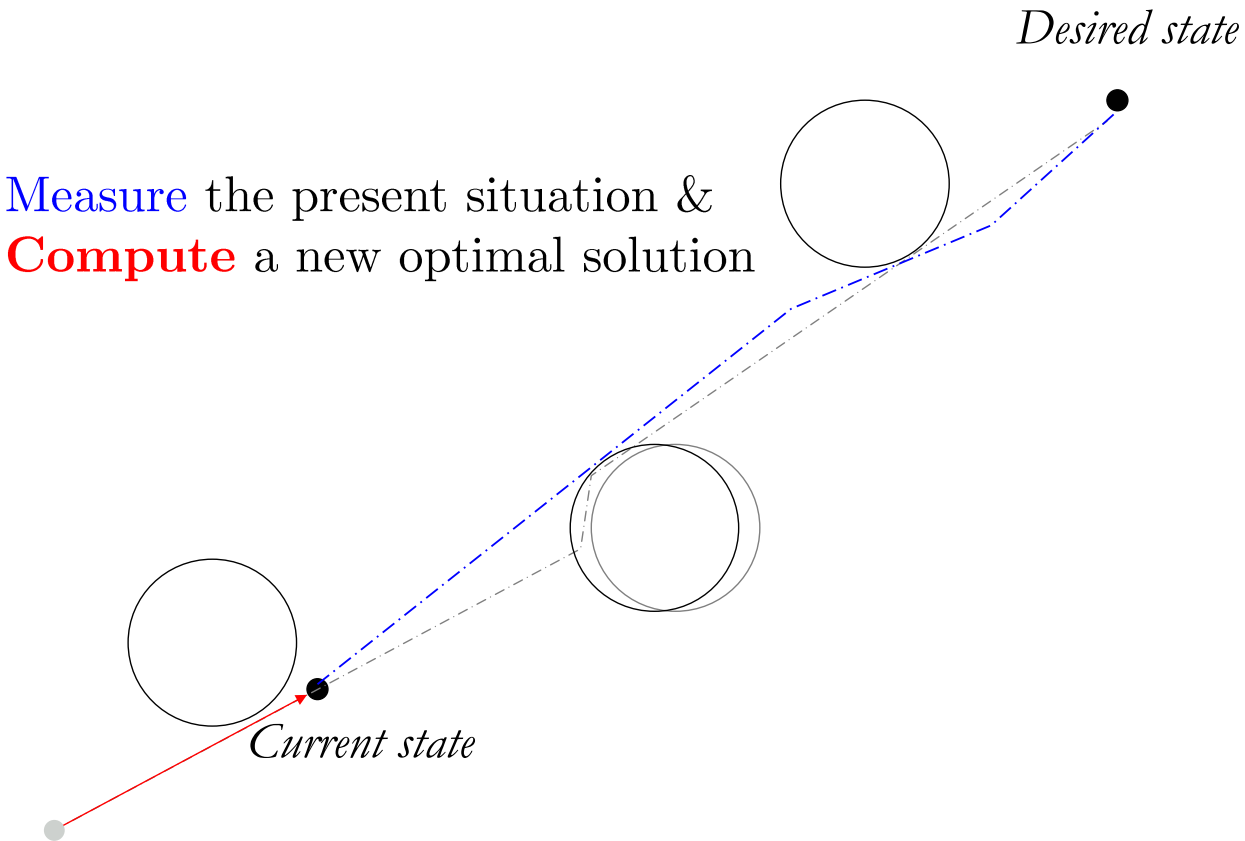

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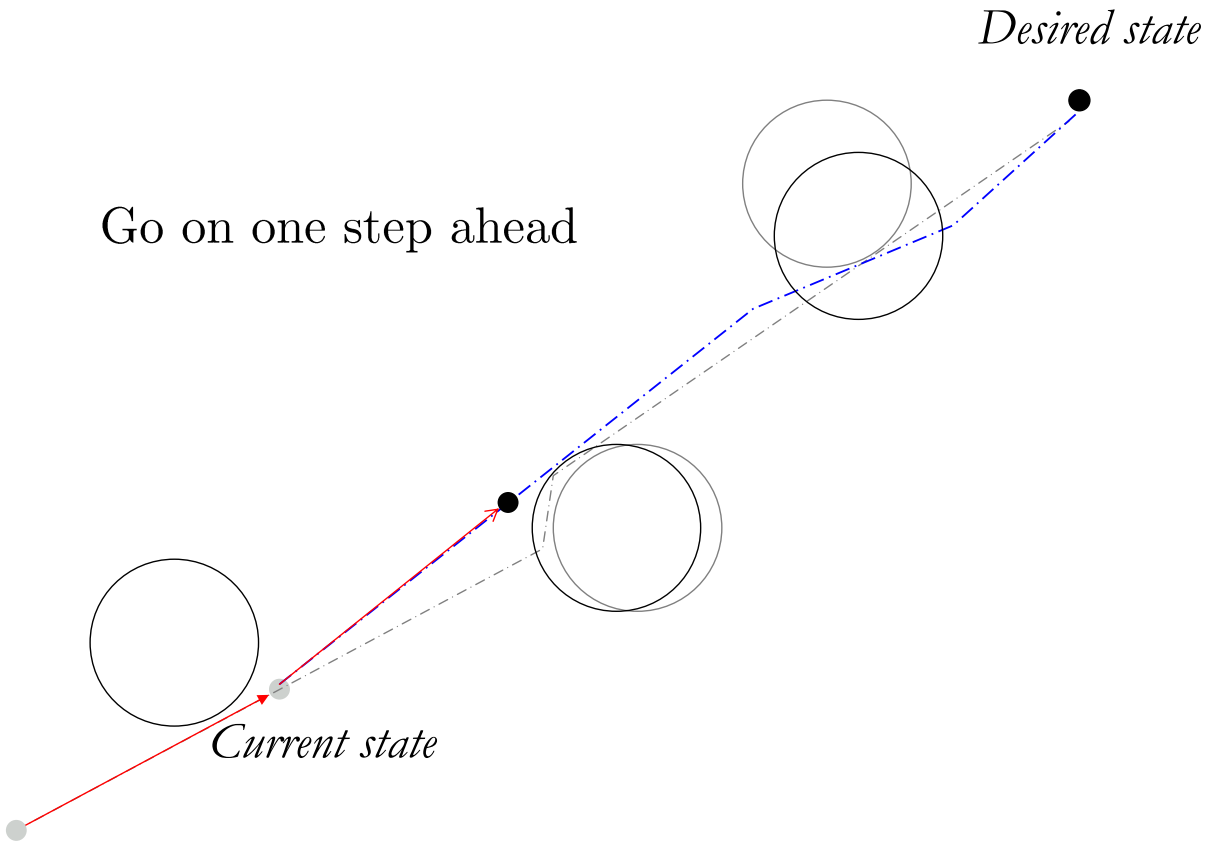

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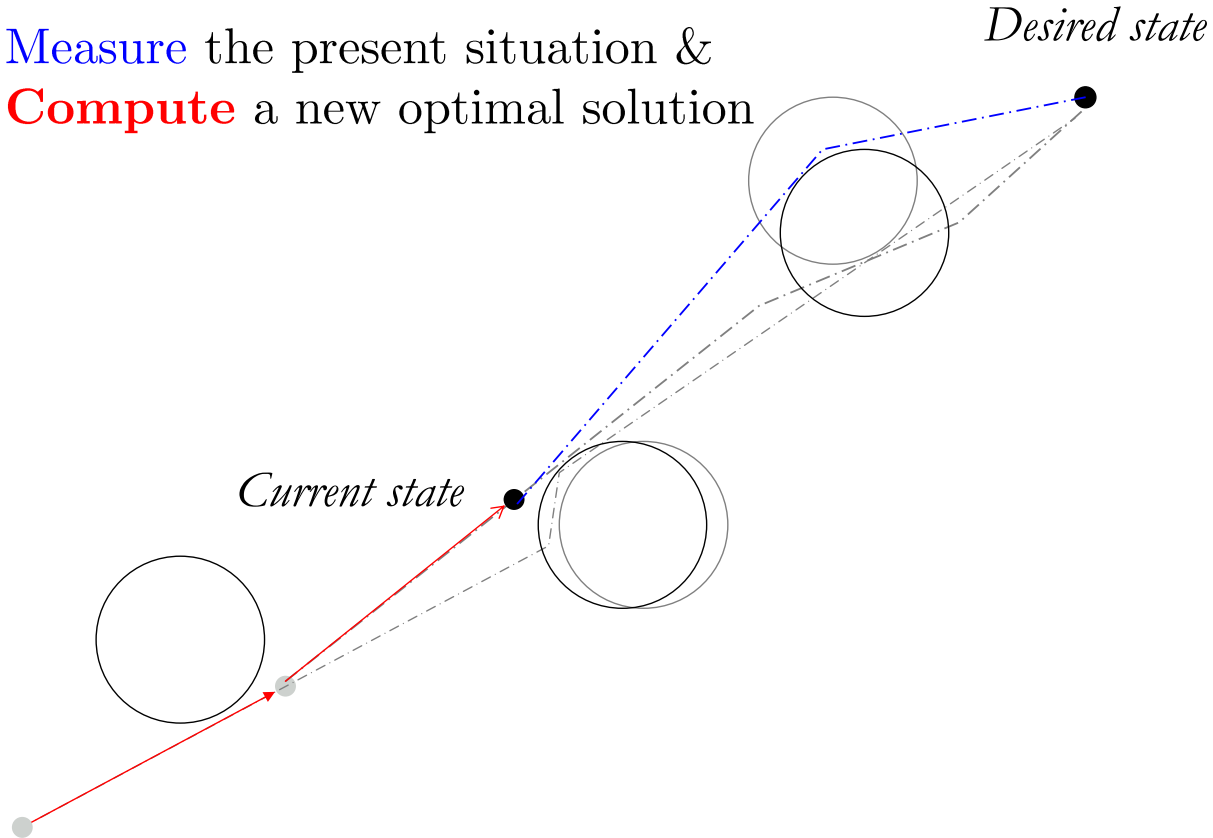

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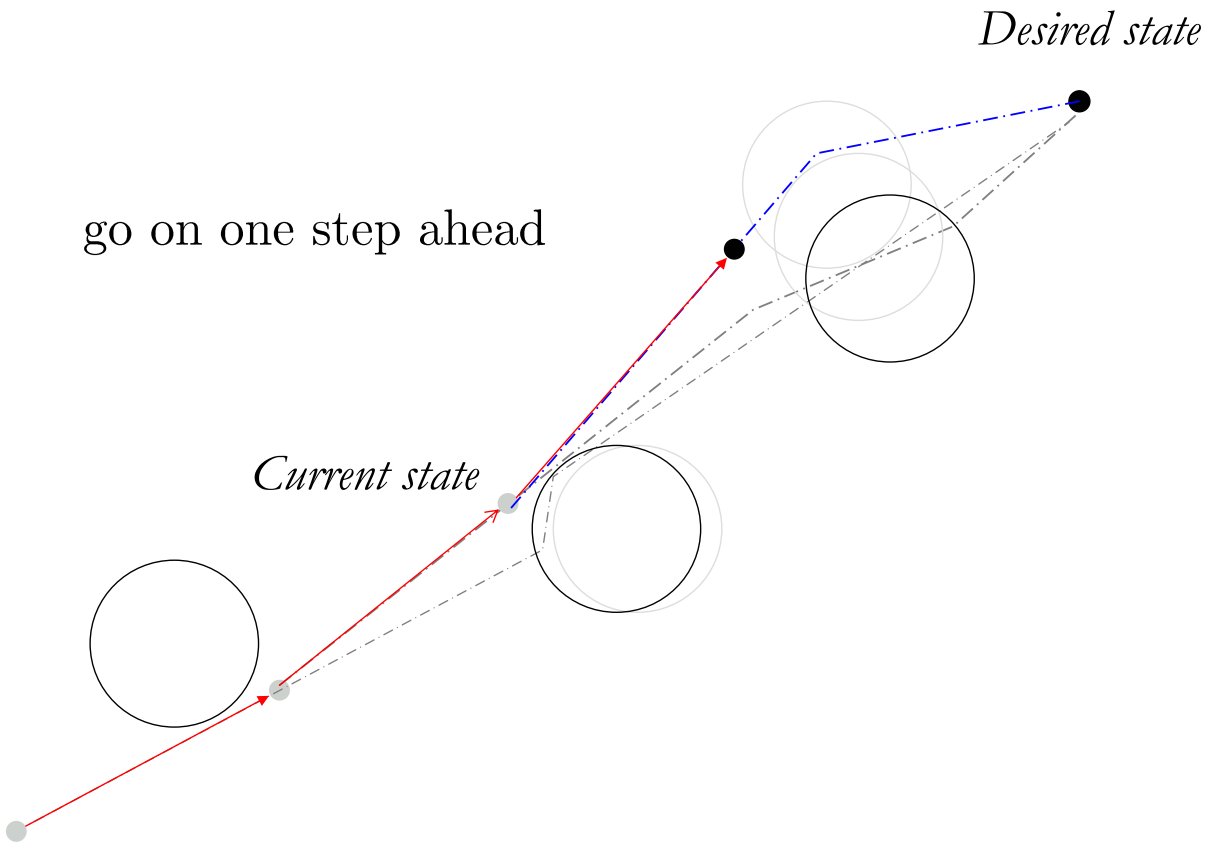

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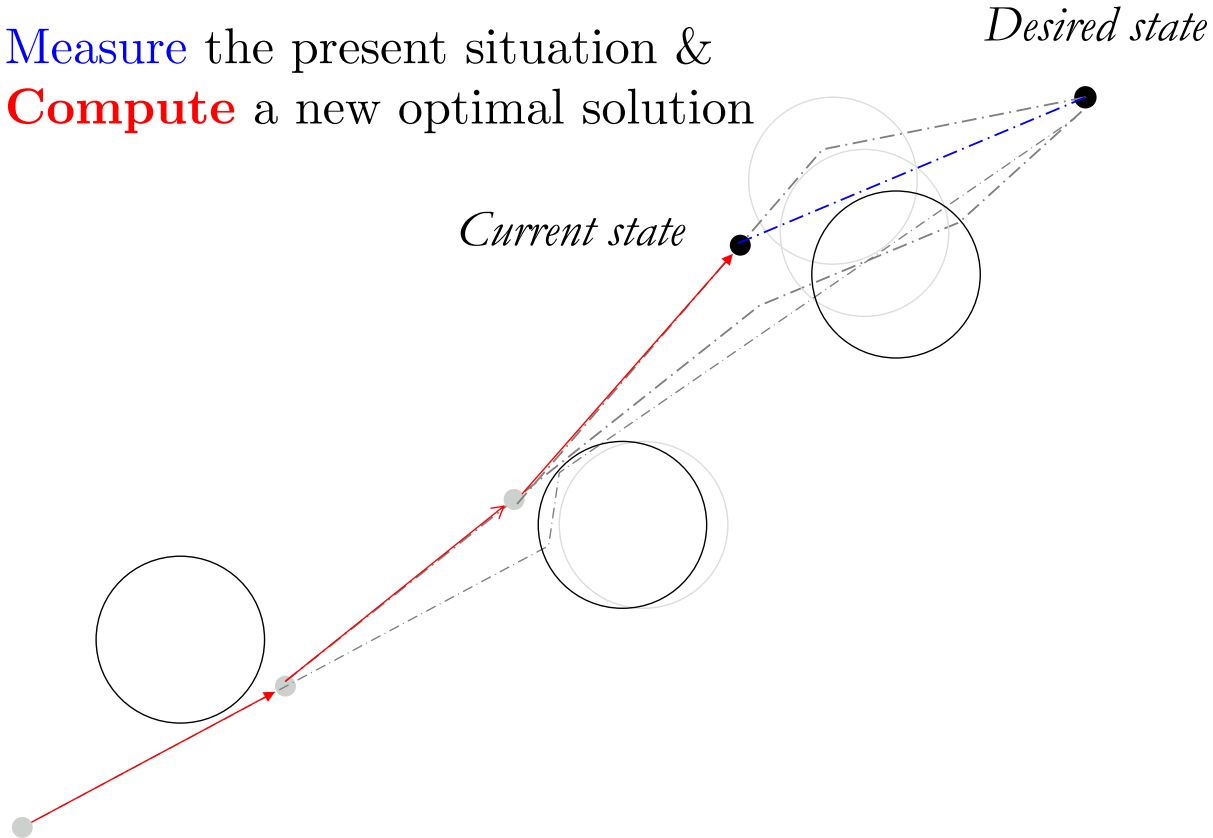

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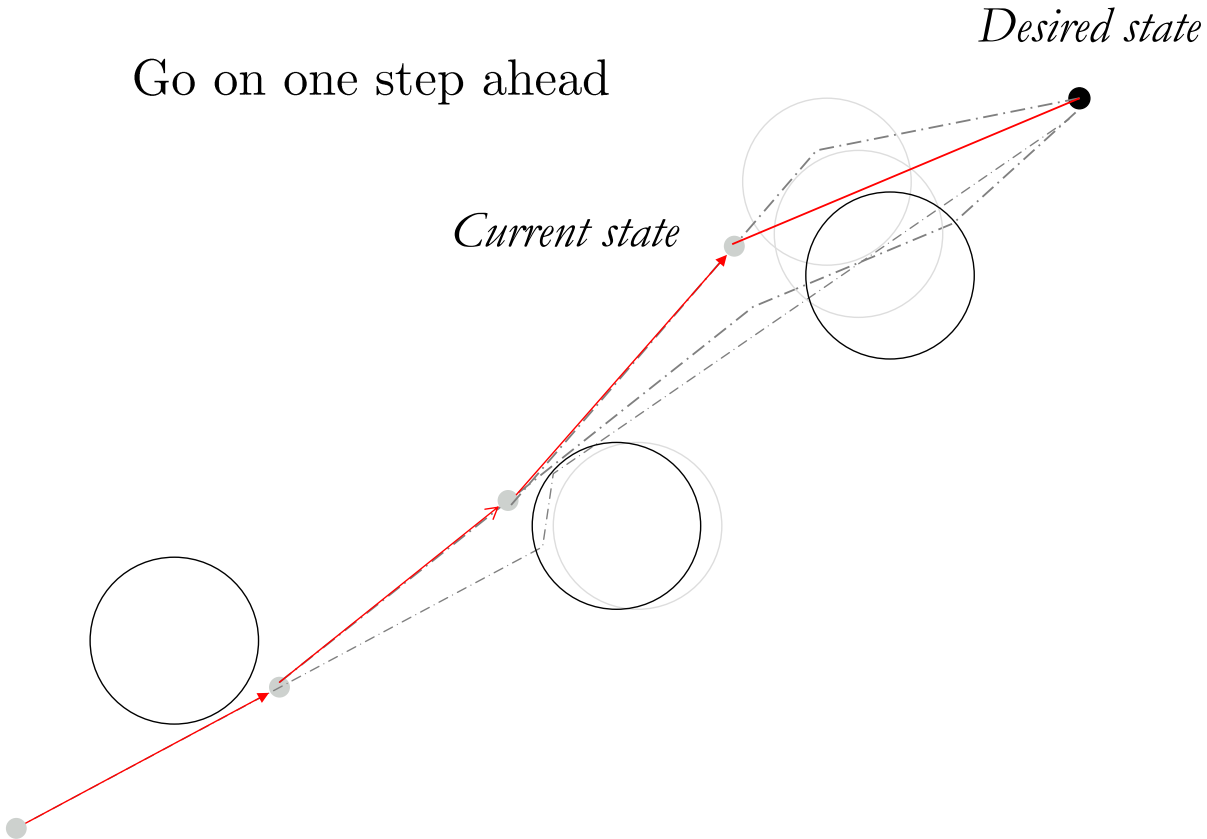

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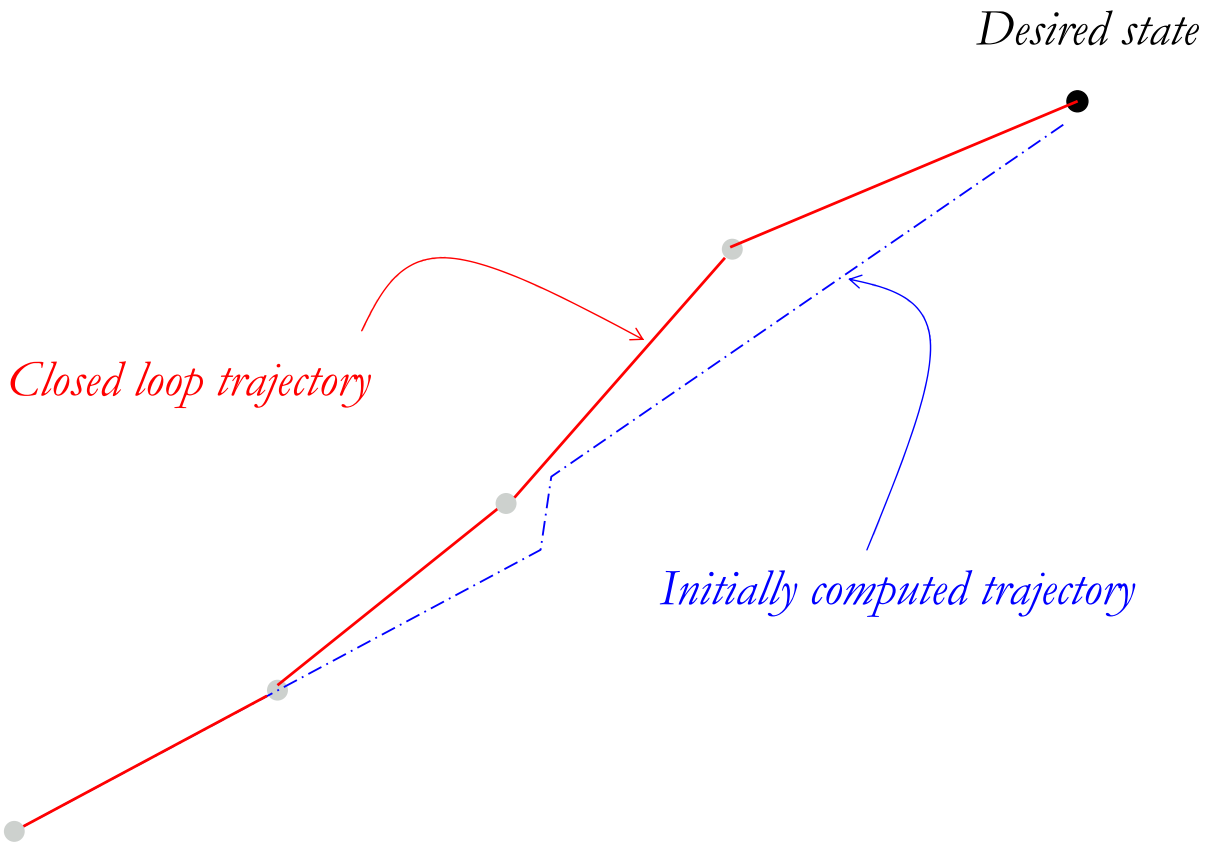

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Consider a dynamical system :

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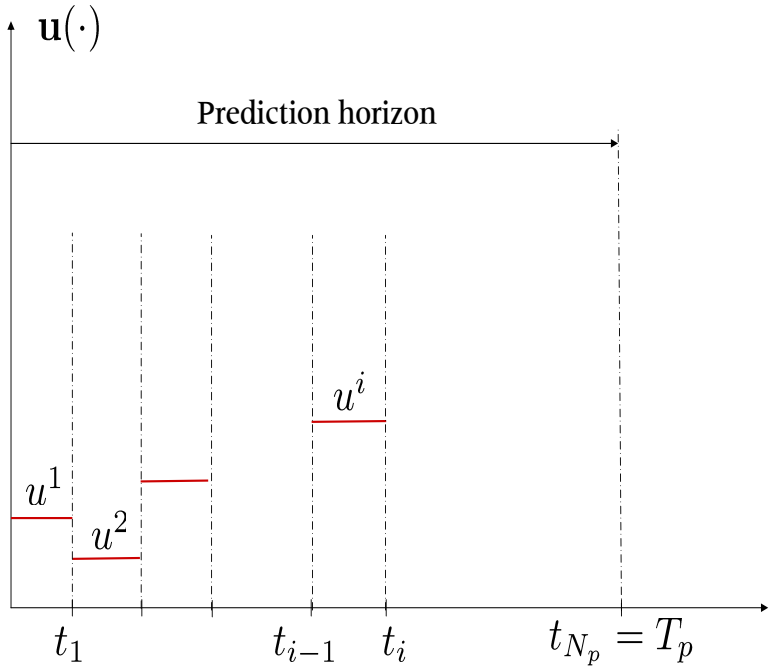

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More generally

Any map

$$C : \mathbb{P} \rightarrow \mathbb{U}^N$$
$$C(p) = \begin{pmatrix} u^1(p) & \dots & u^N(p) \end{pmatrix}$$

defines a $\mathbb{P}$-parametrized piecewise constant control profile

$$\mathbf{u} = \mathcal{U}_{pwc}(\cdot, p)$$

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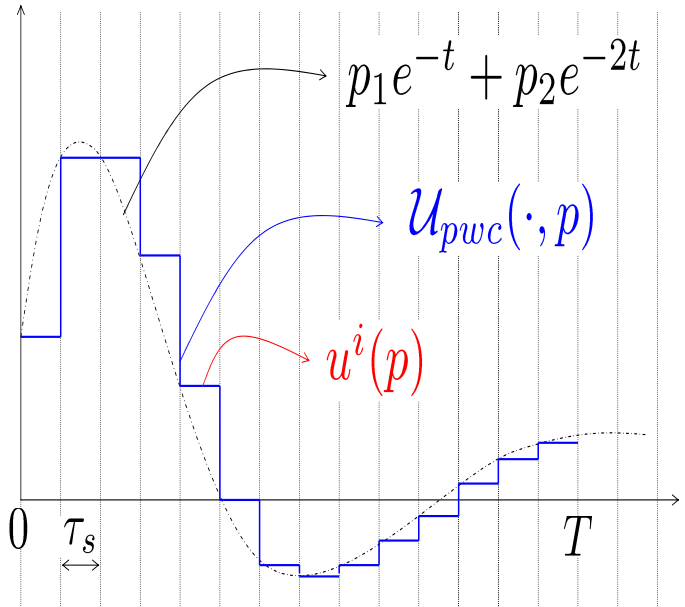

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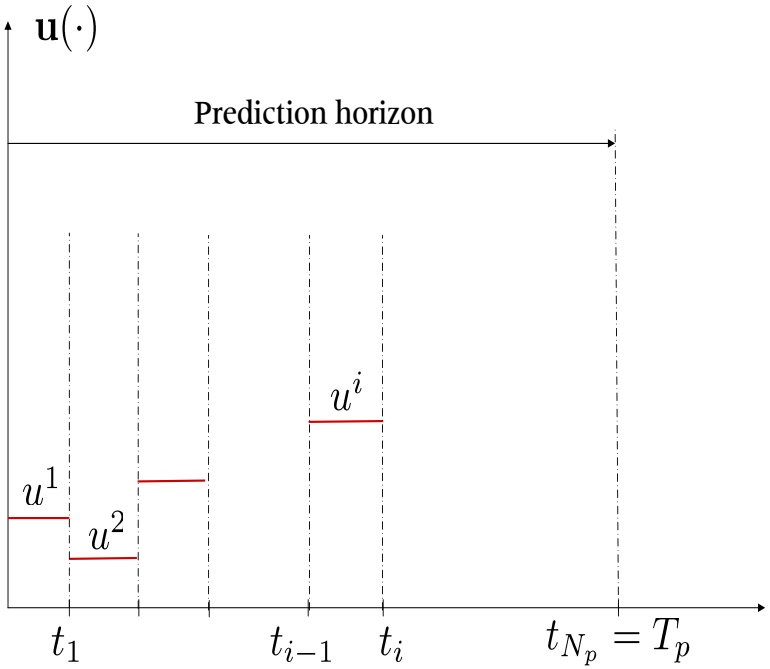

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$$\hat{p}(x(t_j)) := \arg\min_{p \in \mathbb{P}} \Big[J(x(t_j), p) \Big] \quad \text{under} \quad C(x(t_j), p) \leq 0$$

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Advantages

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Price to pay

Solving ON-LINE non convex optimization problems.

MPC is almost a technology . . . (but not NMPC)

Non-exhaustive MPC Vendor List

- ABB
- ACT
- Adaptics
- Adaptive Resources
- Adersa Home Page
- Aspen Technology
- Aurel Systems Inc.
- Batch CAD
- Bonner and Moore
- Brainwave
- C.F. Picou and Associates
- Chemstations
- Comdale Technologies
- Control Arts Inc.
- Control Consulting Inc.
- Control Dynamics Homepage
- Controlsoft Incorporated
- Cybosoft
- DOT Products
- Trieber Controls
- Yokogawa APC
- US Process Control L.L.C.
- Eldridge Engineering Inc.
- Elsag Bailey
- Envision Systems Inc.
- Gensym
- Enterprise Control Technologies
- Fantoft Process Group
- MATHWORKS
- Honeywell
- Hyprotech
- Inferential Control Company
- IntellOpt
- Knowledgescape
- MDC Technology
- Neuralware
- Nexus Engineering
- Objectspace
- Optimal Control Research
- Pavilion Technologies
- Predictive Control Ltd.
- Process System Consultants
- RSI
- Simulation and Advanced Controls Inc.
- Simtech
- Texas Controls Inc.

Typically linear applications and tools

R. Findeisen: NMPC for Fast Nonlinear Systems (Workshop CDC 2006, San Diego)

The stability is an issue !

Consider the dynamical system

$$\begin{aligned} x_1^+ &= x_1 + (1 + x_2^2)u \\ x_2^+ &= \frac{3}{2}x_2 - x_1 e^u \end{aligned}$$

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- ✓ Open-loop instable.
- ✓ Set of equilibrium states

$$\mathcal{E}_{st} = \left\{ x^{(\alpha)} := \begin{pmatrix} \alpha \\ 2\alpha \end{pmatrix} \quad ; \quad \alpha \in \mathbb{R} \right\}$$

Control objective starting at $x^{(0)} = (0, 0)$, stabilize the system around $x^{(1)} = (1, 2)$.

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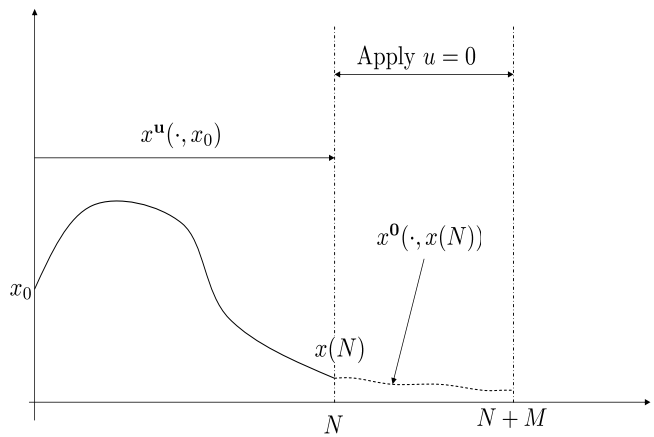
Consider the cost function

$$J(x, \mathbf{u}) := F(x(N)) + \sum_{i=0}^{N} L(x(i), u(i))$$

where

$$L(x, u) := \|x - x^{(1)}\|^2 + ru^2$$

$$F(x) = \sum_{i=1}^{M-N} x^{\mathbf{0}}(i; x)$$



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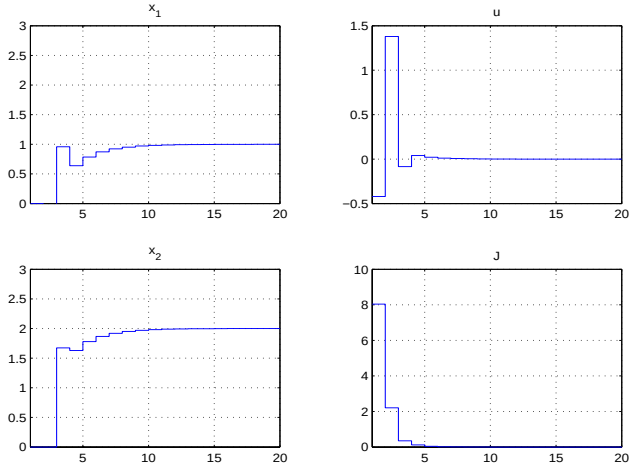

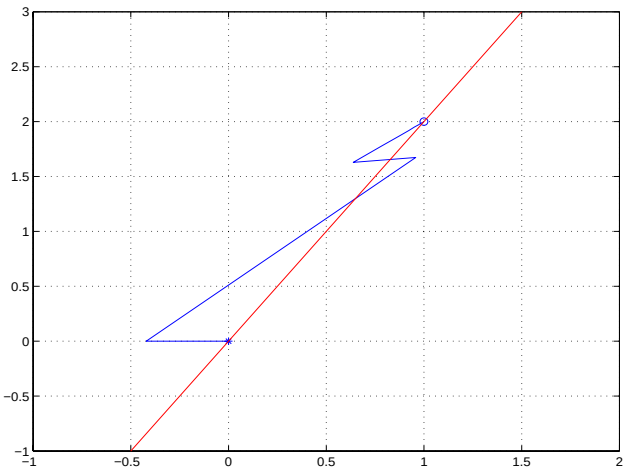

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Let us assume that one looks for solution such that $u \leq 0.1$

Assume that for that reason one takes

$$r = 160$$

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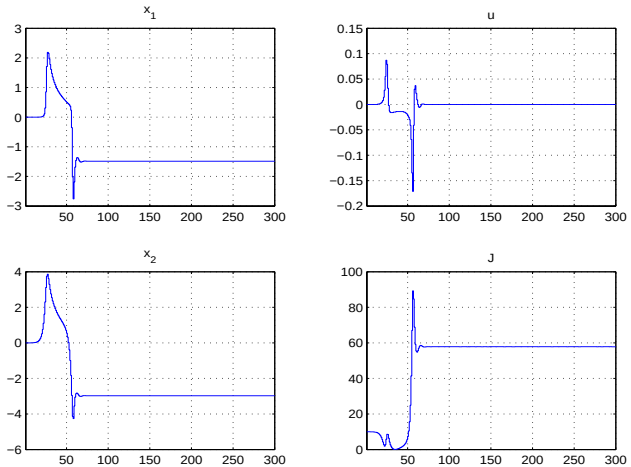

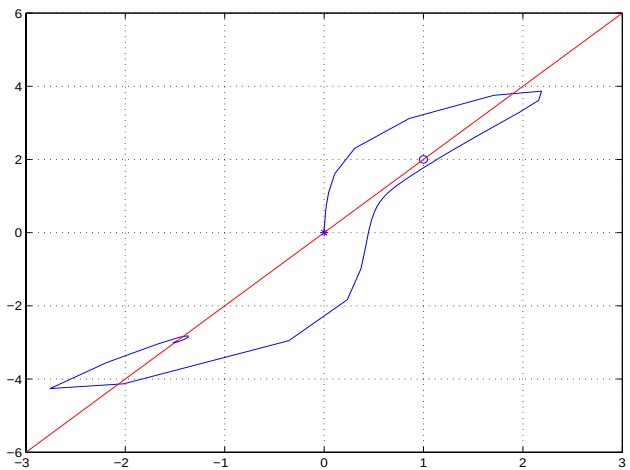

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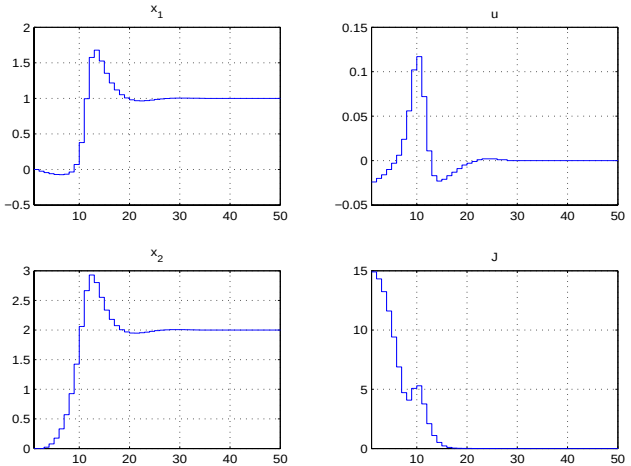

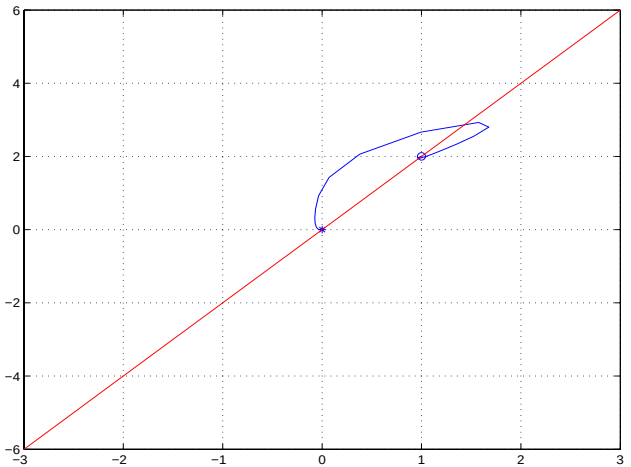

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However, the control is again above 0.1,

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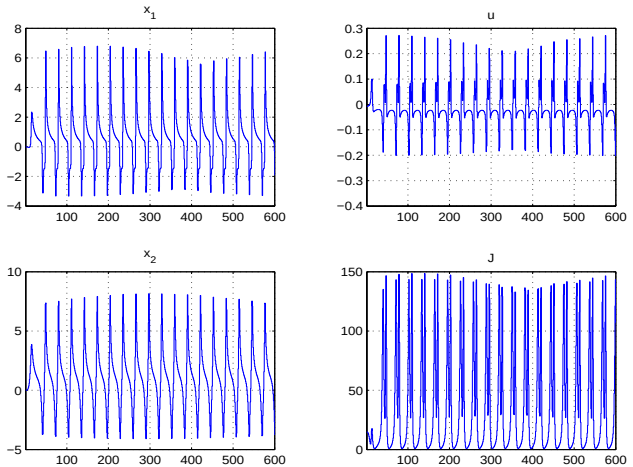

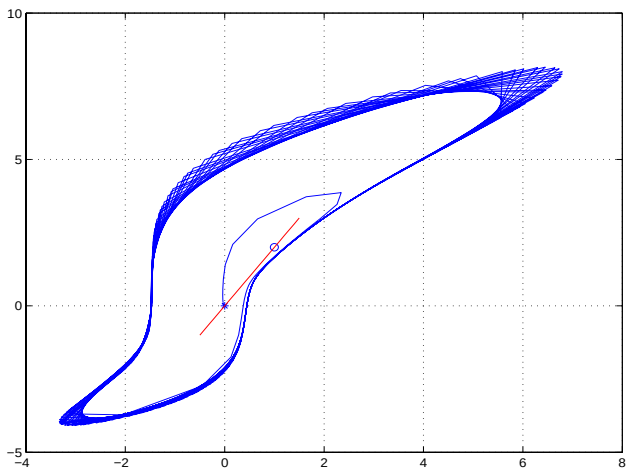

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Let us explicitly impose the constraint

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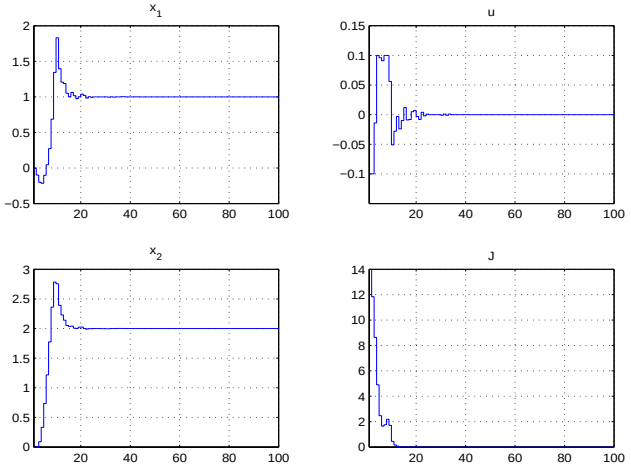

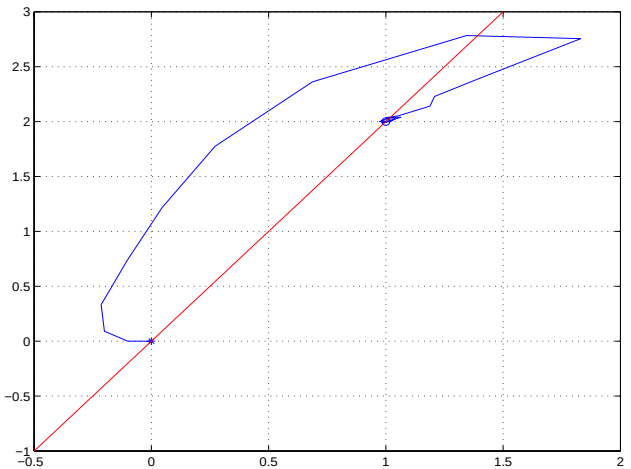

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- For a detailed study of the stability conditions

  Mayne,D.Q., Rawlings,J.B., Rao,C.V., et al , Constrained model predictive control : stability and optimality, Automatica, 2000, Vol : 36, Pages : 789 - 814

The concept of distributed-on-time optimization

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The computation of $\hat{p}(x(t_k))$ is done using some iterations

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or using some stopping $\varepsilon > 0$:

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where $\imath_\varepsilon(x(t_k)) \in \mathbb{N}$ is the number of iterations that generally depend on $x(t_k)$.

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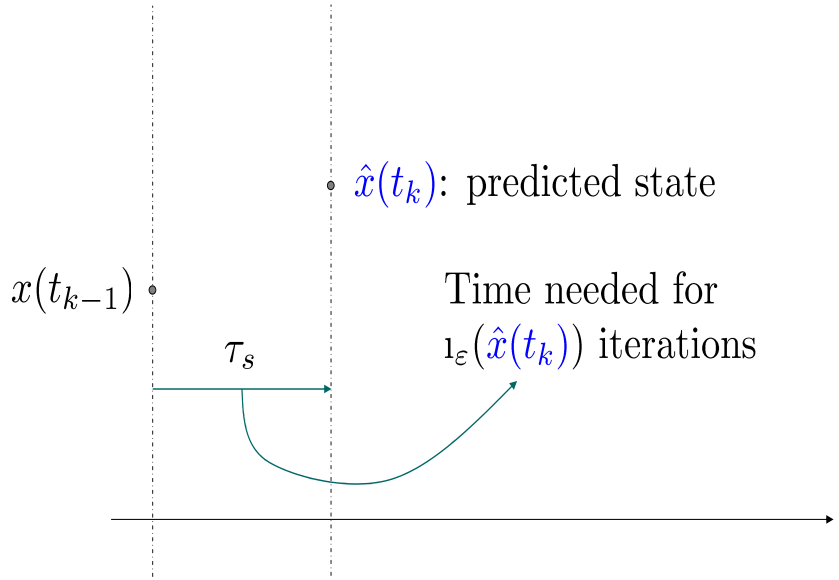

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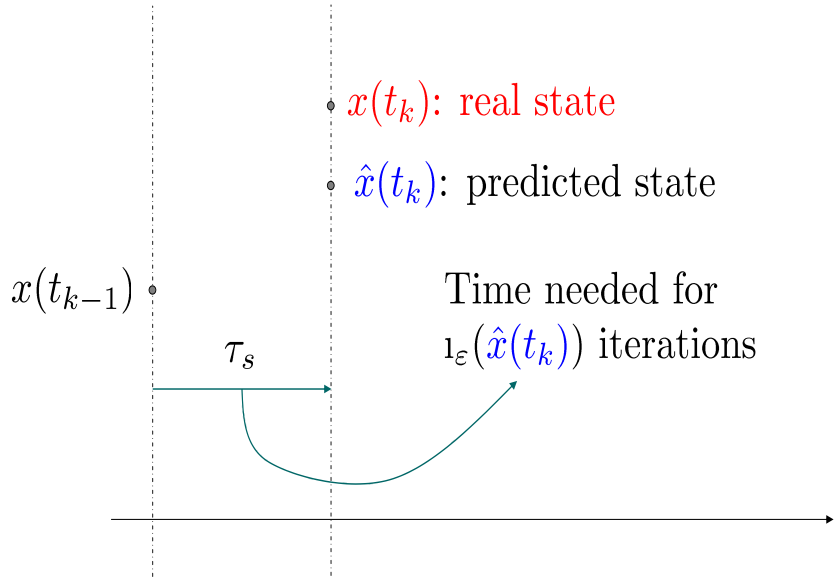

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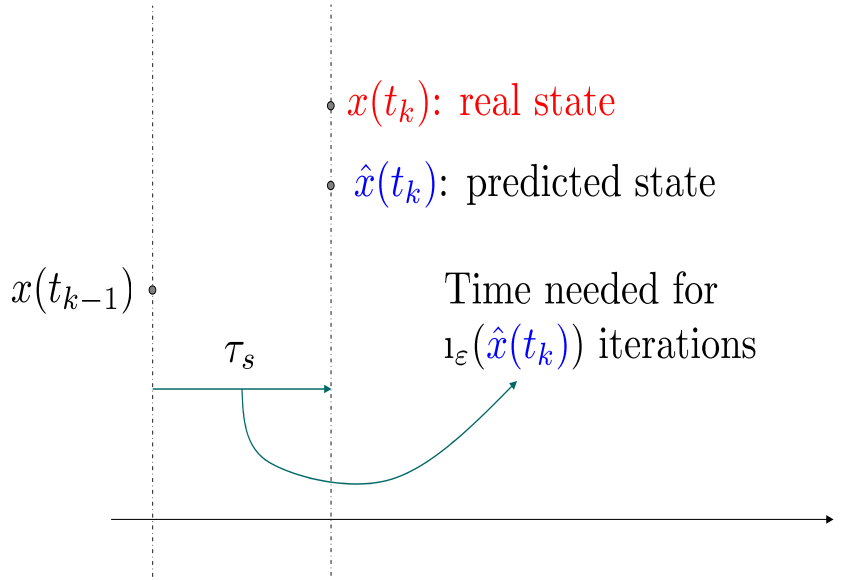

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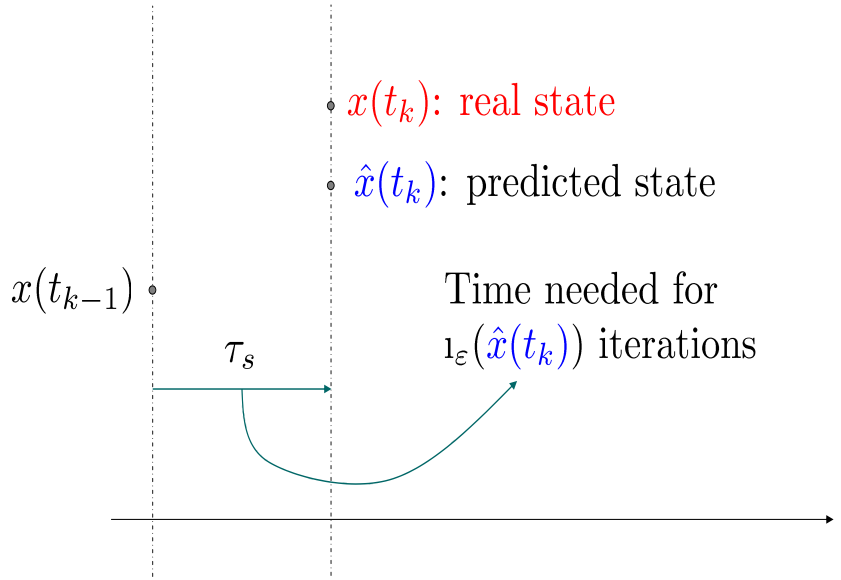

- $\tau_s \geq \imath_\varepsilon(x(t_k)) \times \tau_{\text{iteraton}}$
- $\|x(t_k) - \hat{x}(t_k)\|$ increase with τ_s due to model uncertainties & disturbances

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- Distribute the optimization over the system real life-time :

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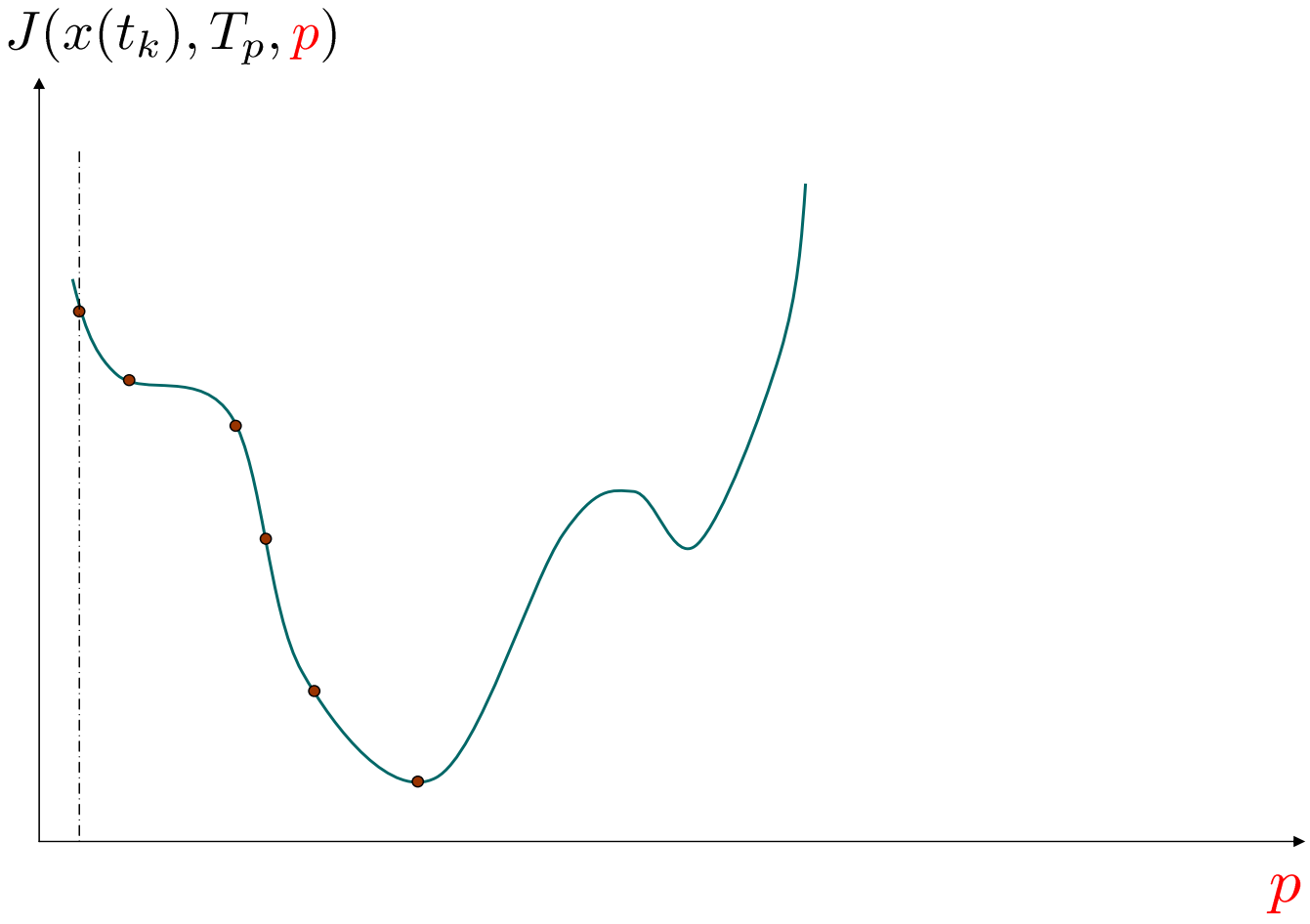
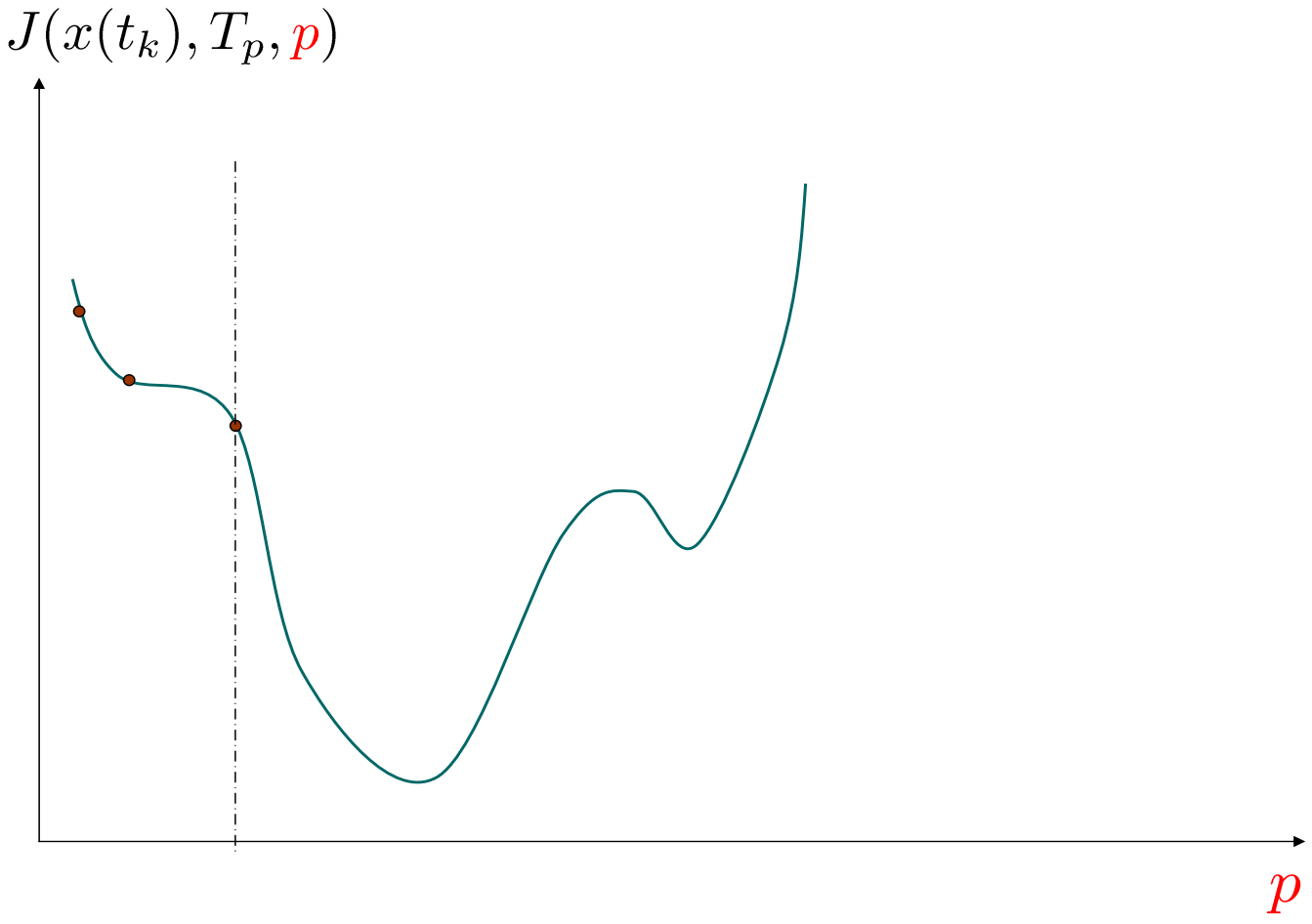
Distributing the optimization over the system real-life time



Case $q = 2$

Distributing the optimization over the system real-life time



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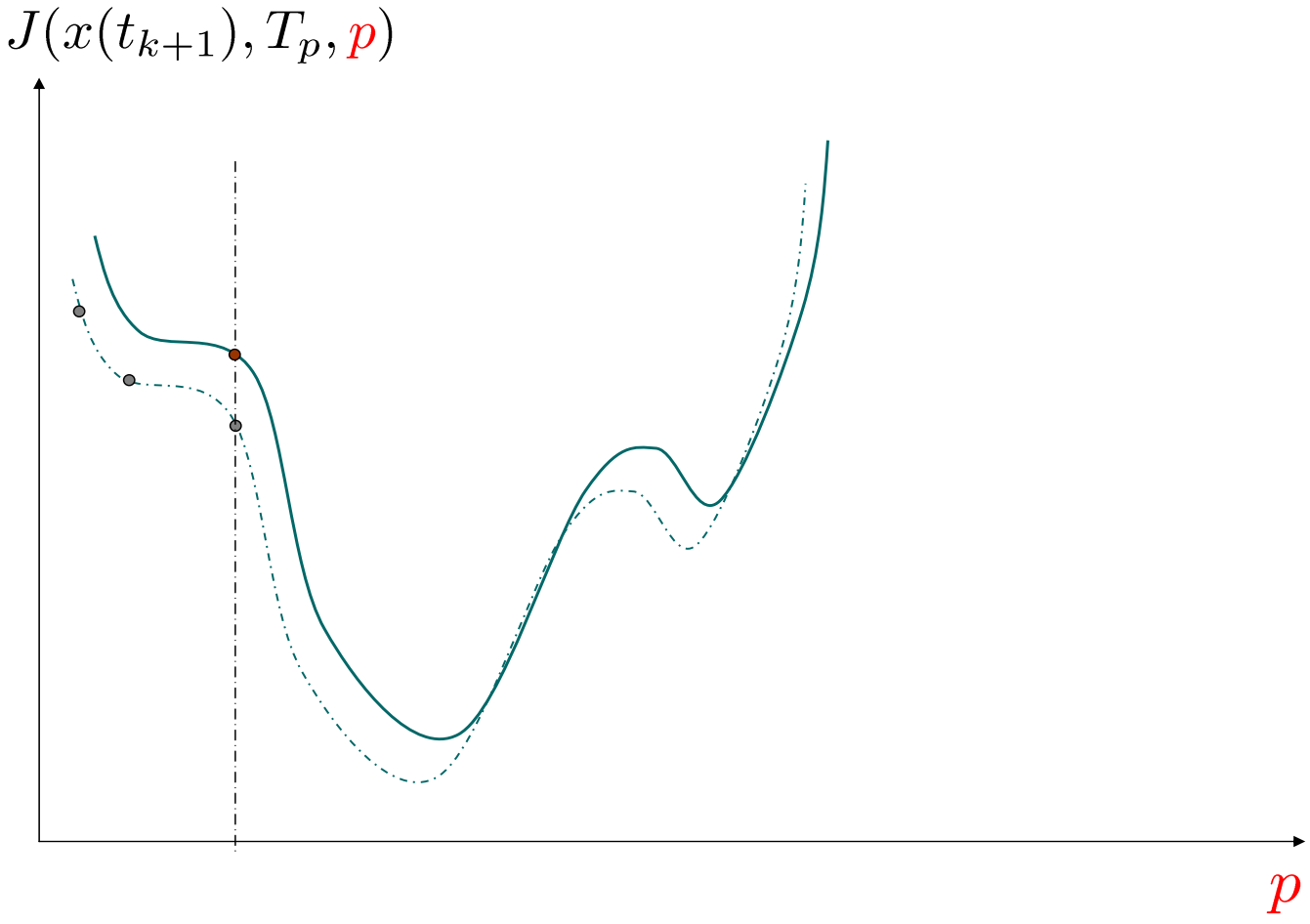

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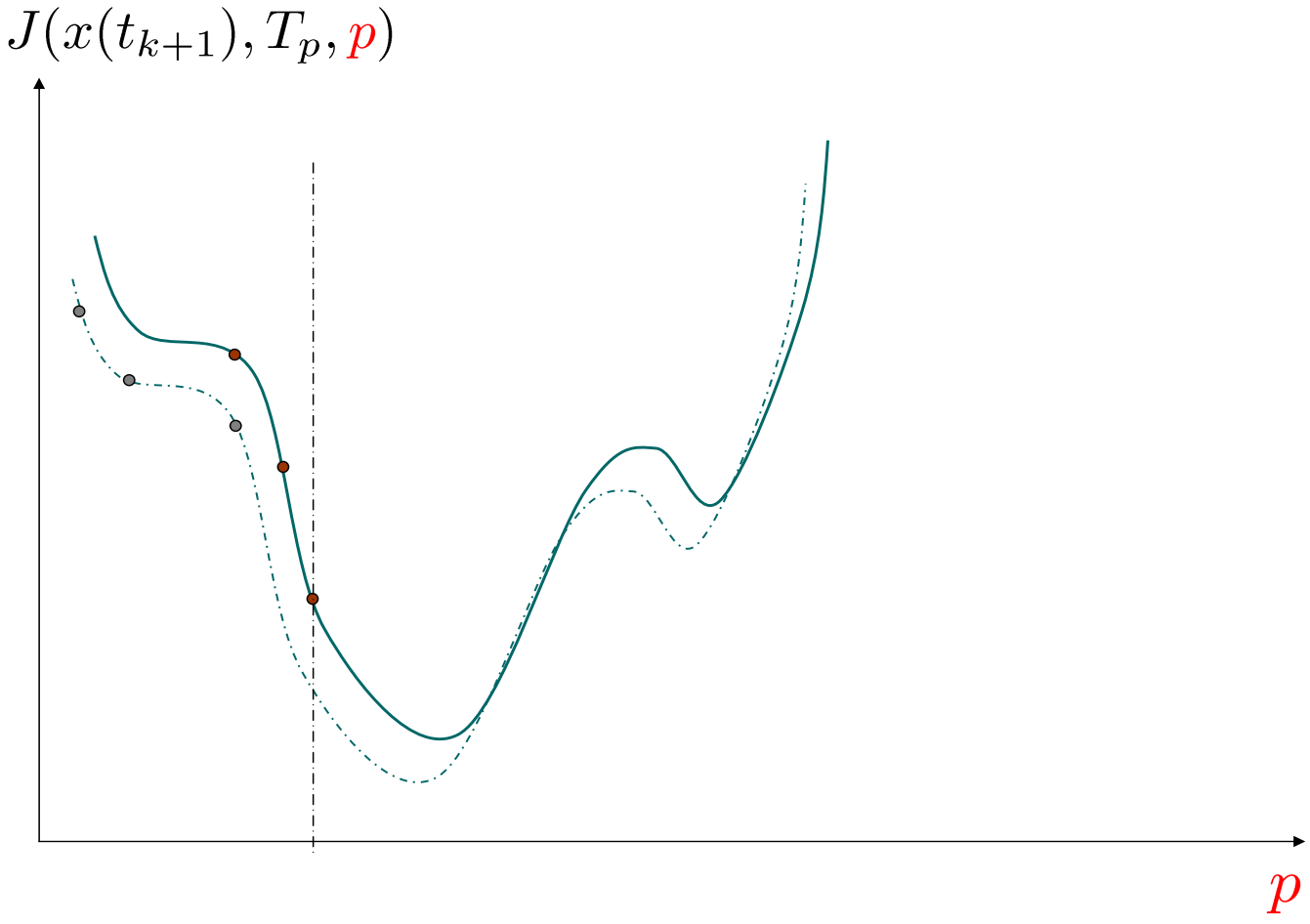

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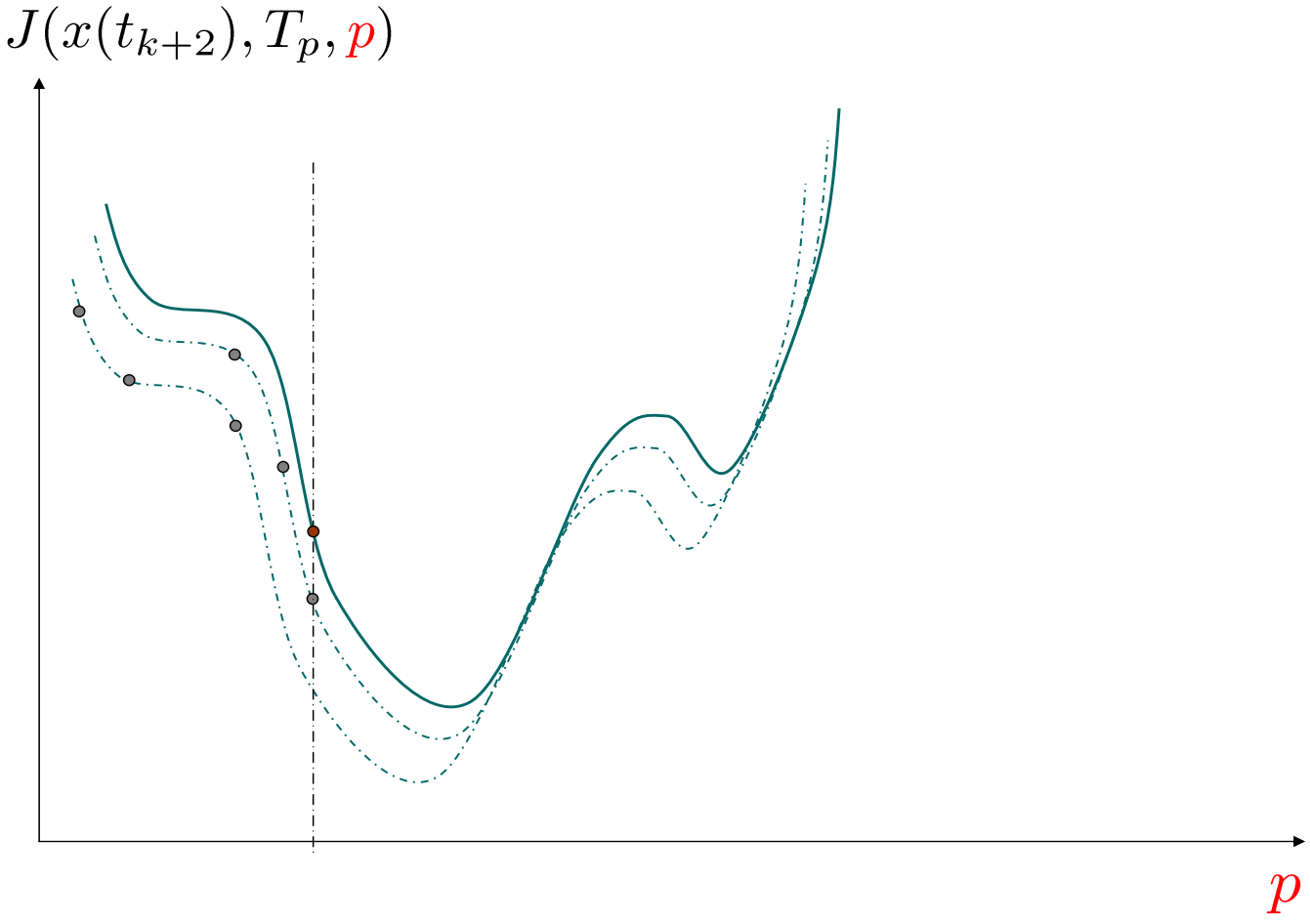

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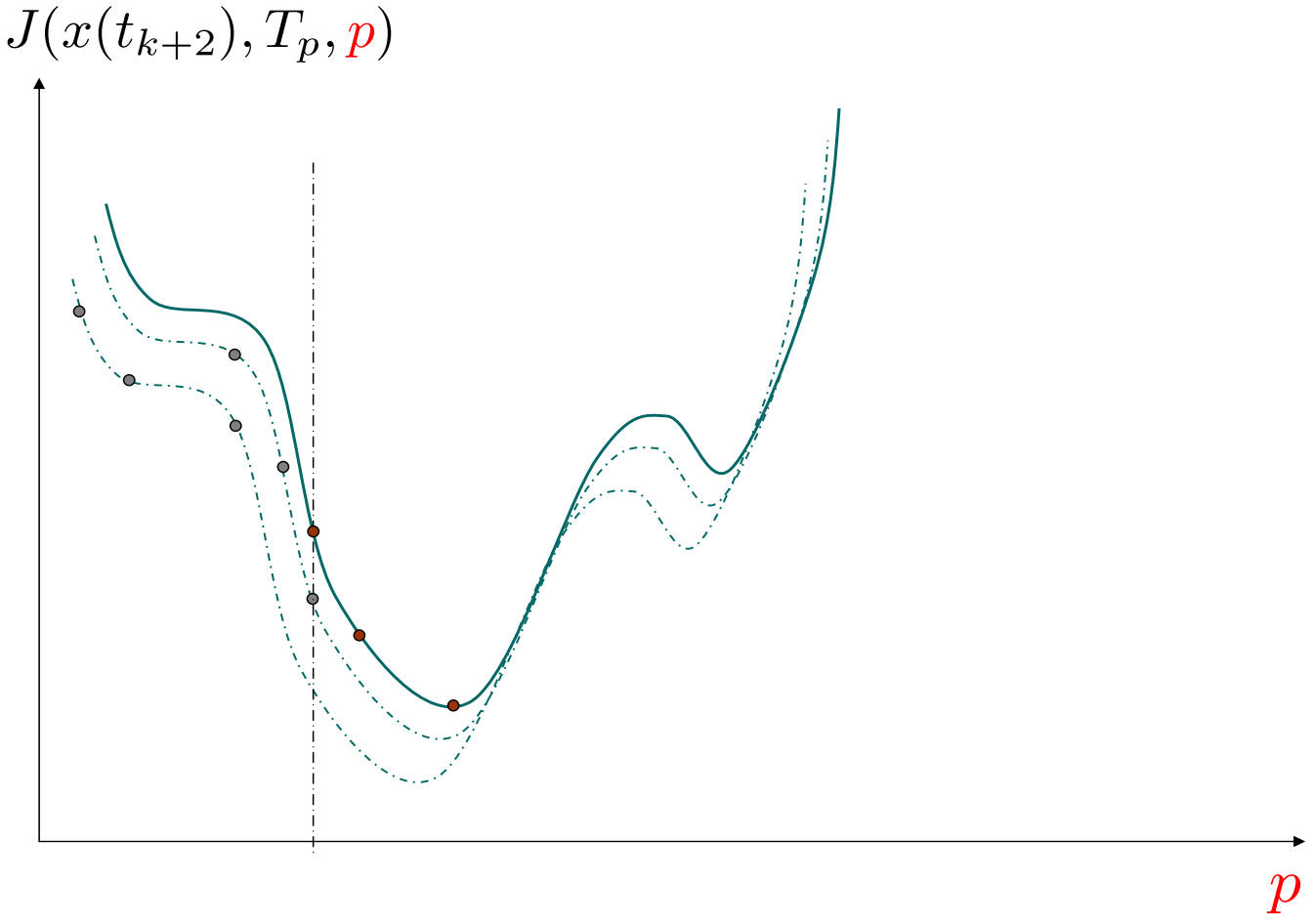

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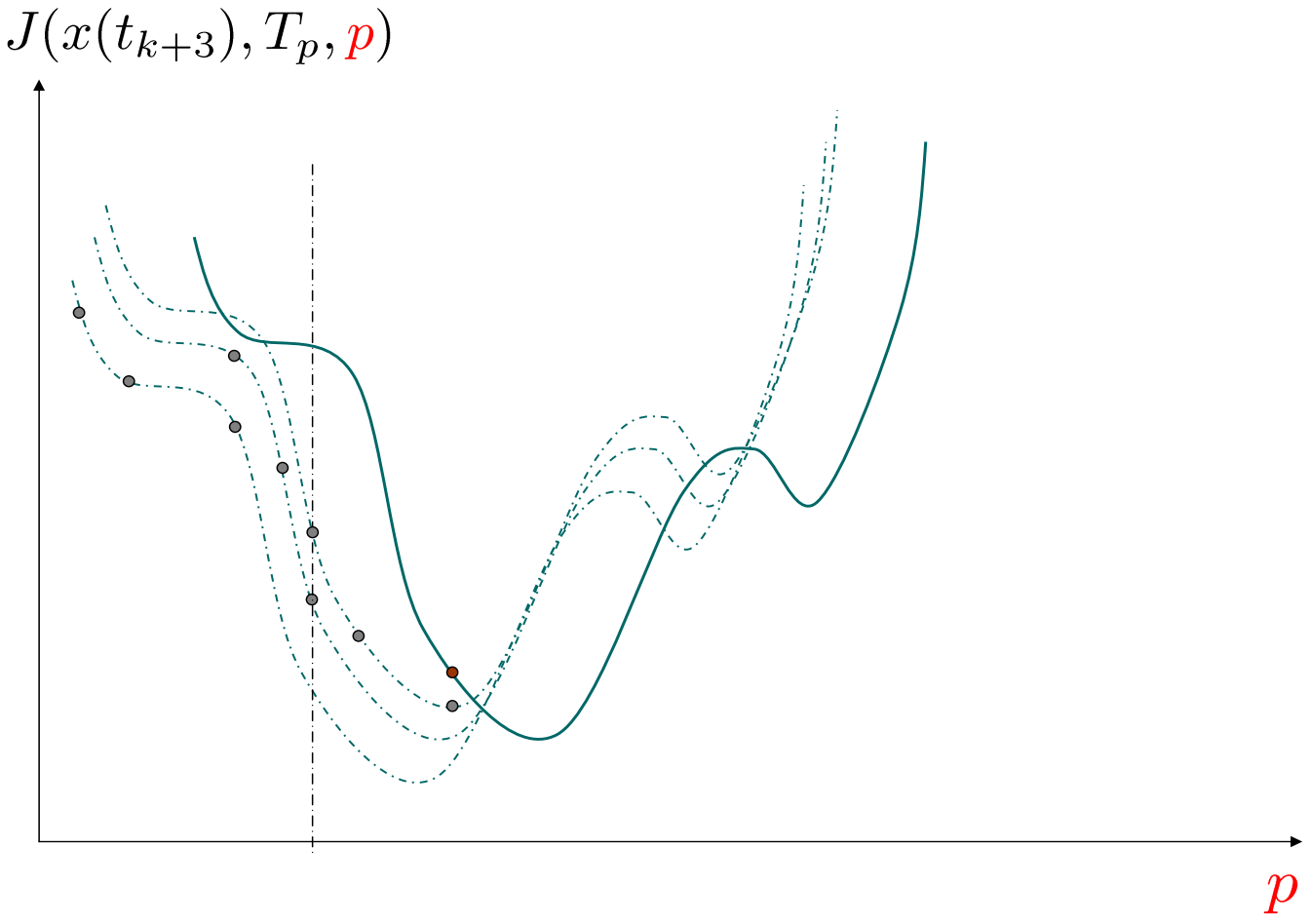

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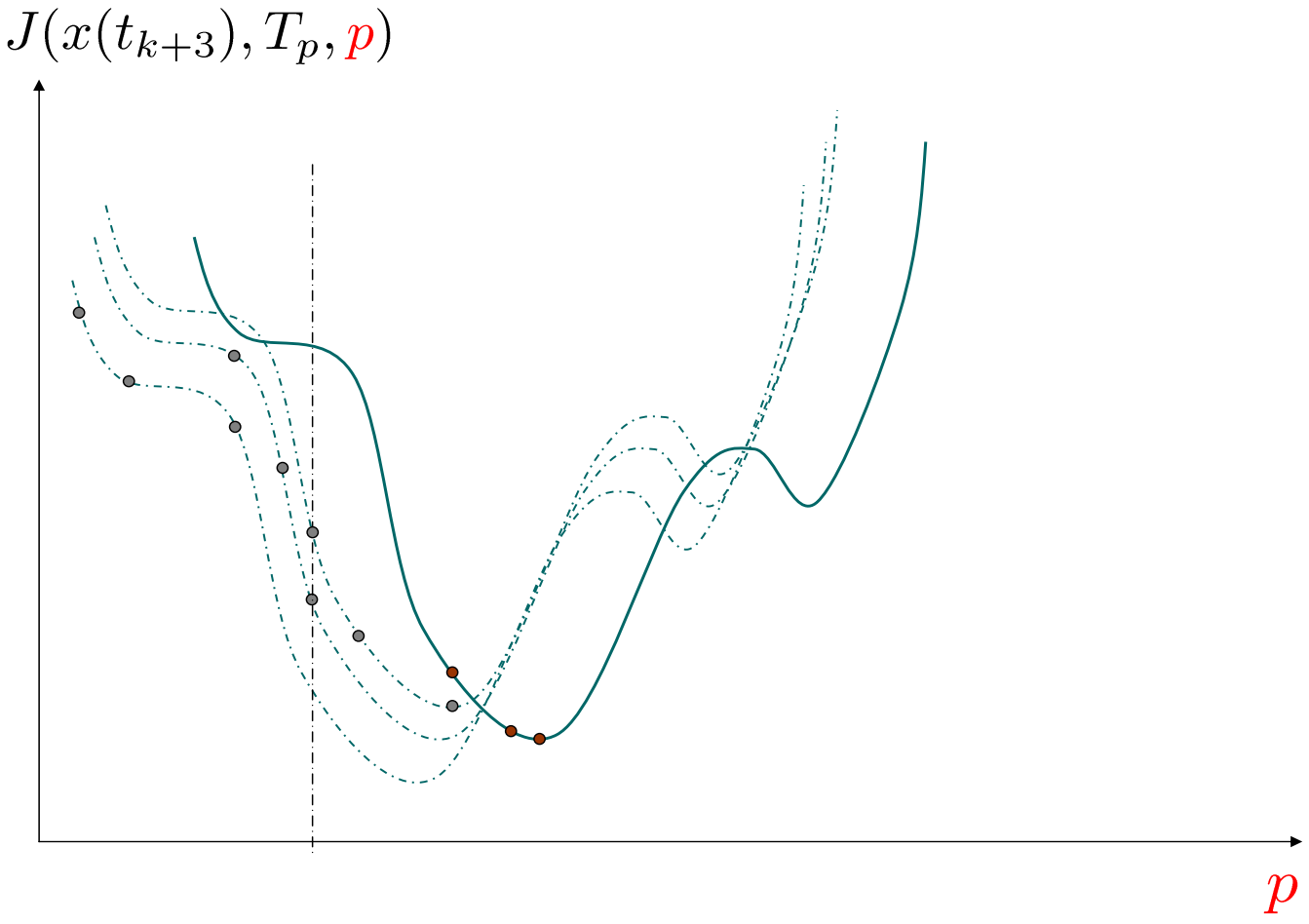

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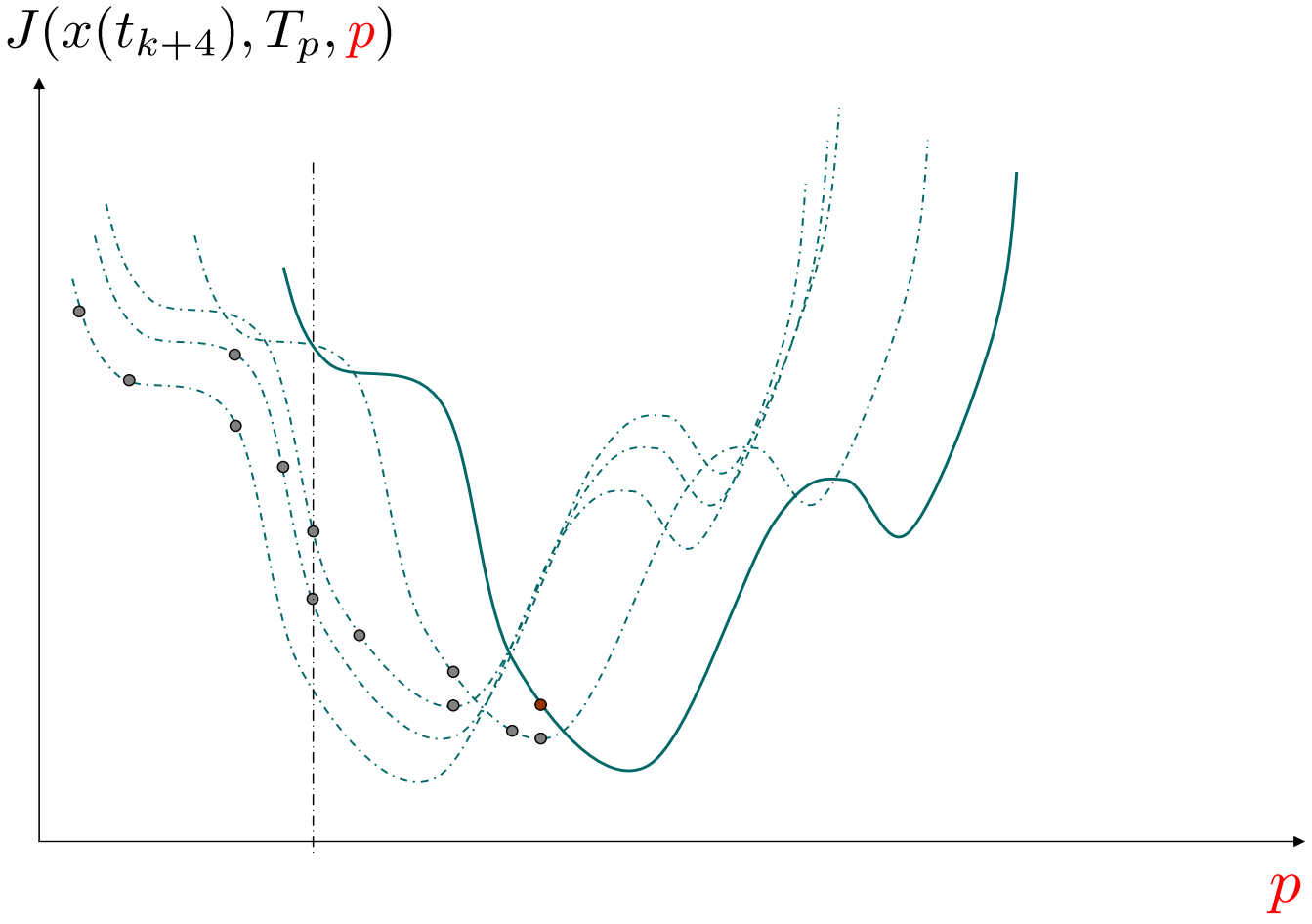

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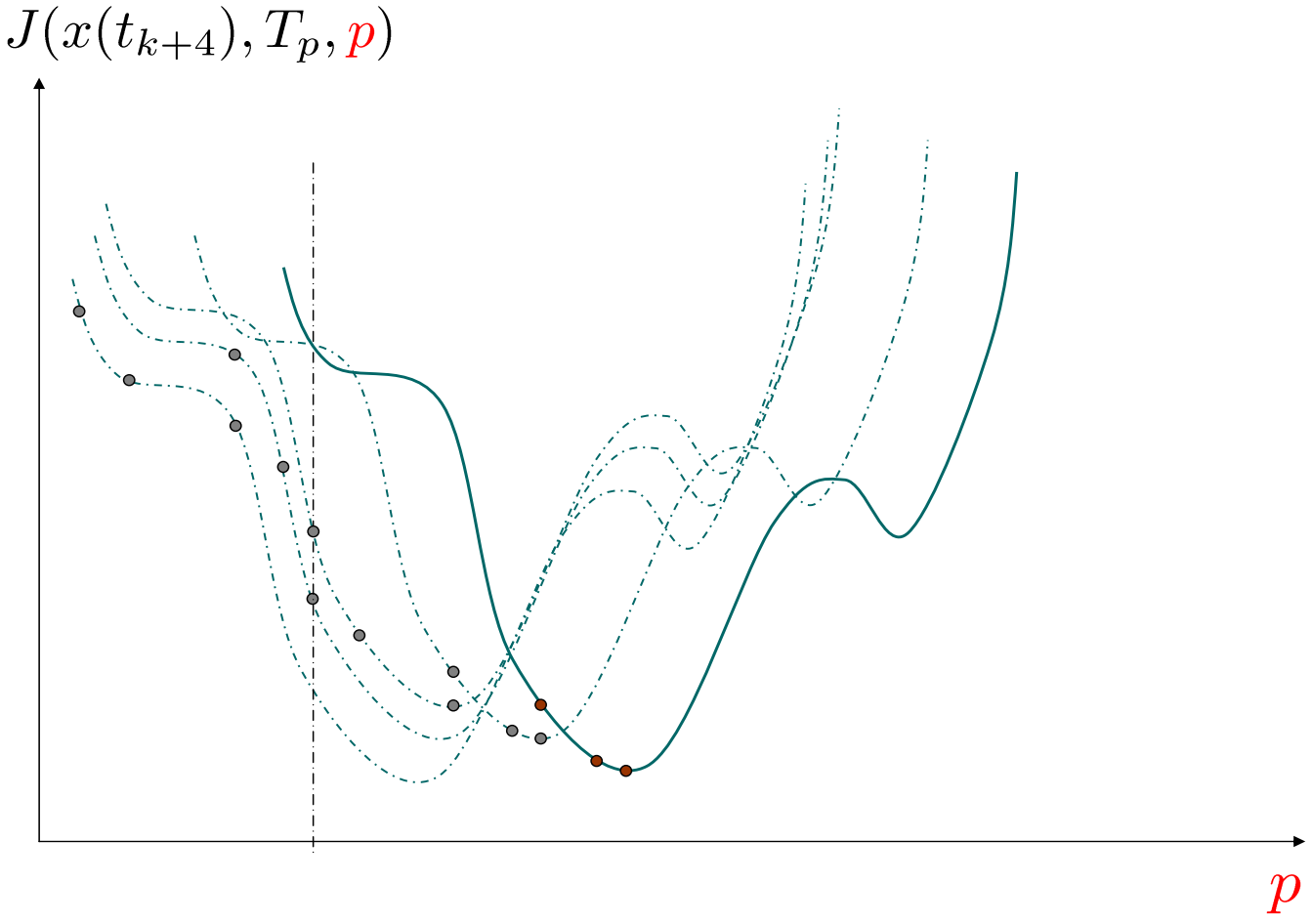

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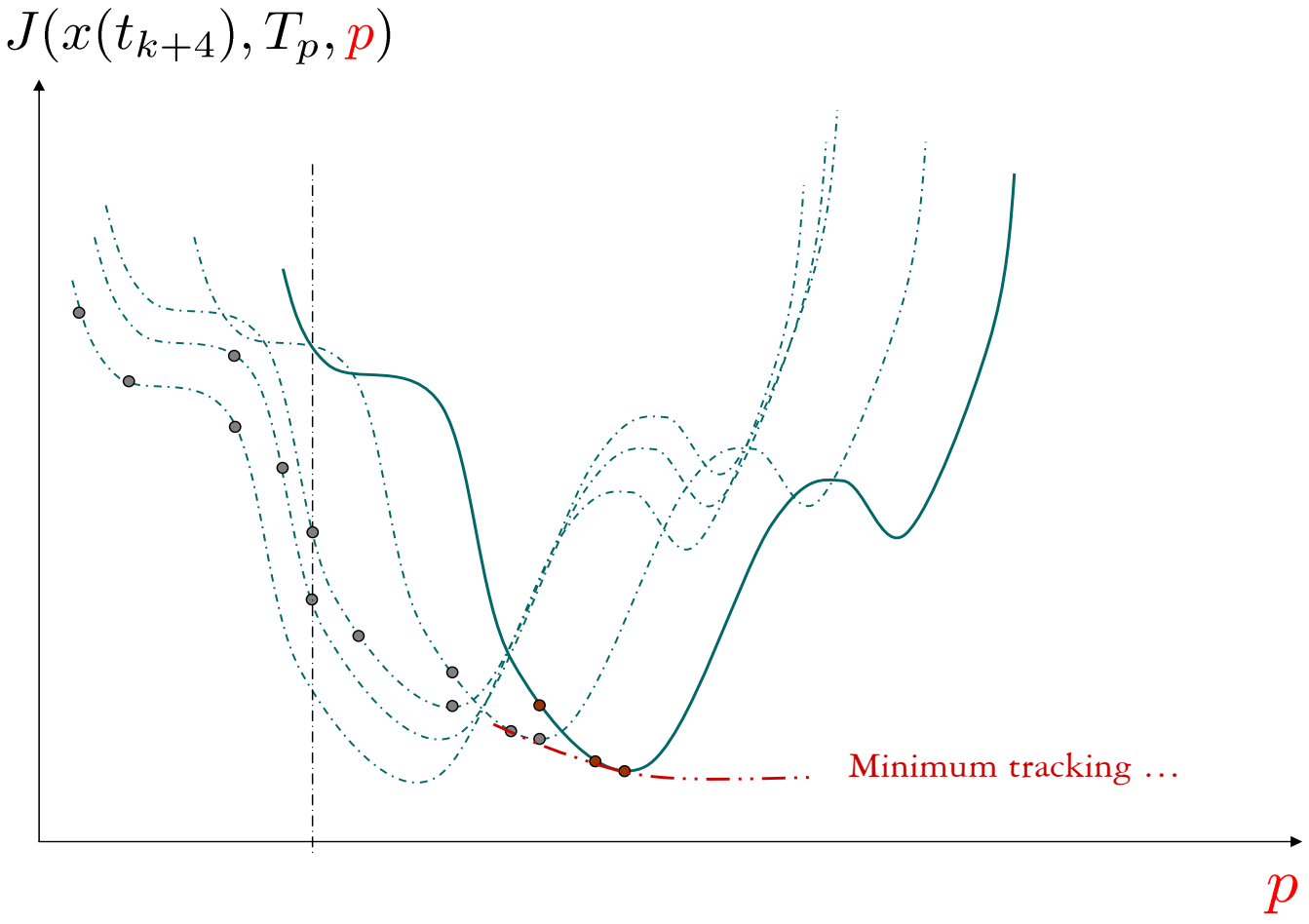

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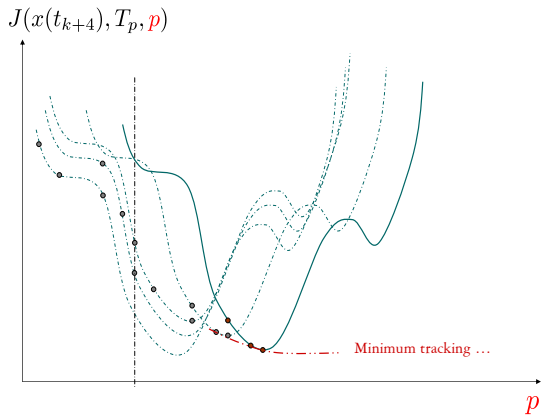

This may work provided that

1. $\tau_s \ll$ characteristic time of the system
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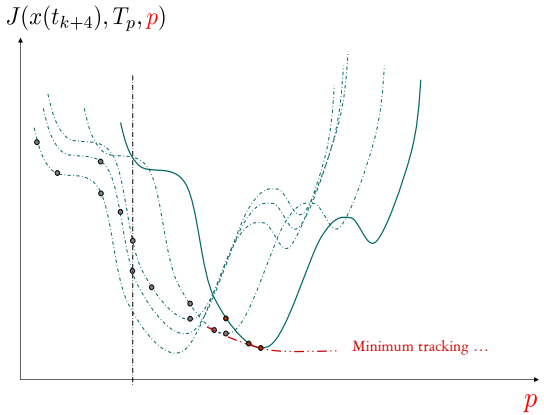

[M.Alamir, A Framework for updating control period monitoring in real-time NMPC schemes International Workshop on Assessment and Future Directions of Nonlinear Model Predictive Control, Pavia, Italy, (2008).]

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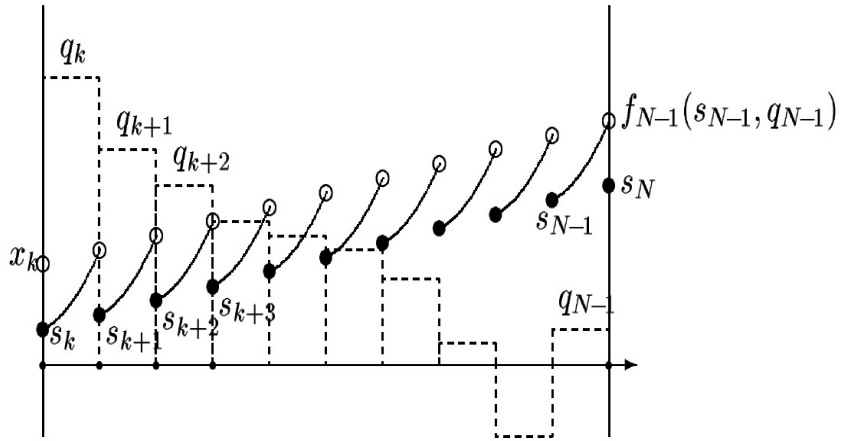
Main Alternatives

The iterative process $\mathcal{S}$

$$\begin{aligned} p^{(i+1)} &= \mathcal{S}(p^{(i)}, x(t_k)) \\ p^{(0)} &= p_0 \in \mathbb{P} \end{aligned}$$

Multiple shooting approach	[Diehl et al. 2005]
Continuation approach	[Ohtsuka 2004]
Parametric approach	[Alamir 2006]

The multiple shooting approach [Diehl et al. 2005]



Unknown

$$y = (\lambda_k, s_k, u_k, \ldots, \lambda_{k+N}, s_{k+N}, u_{k+N})^T$$

Constraints

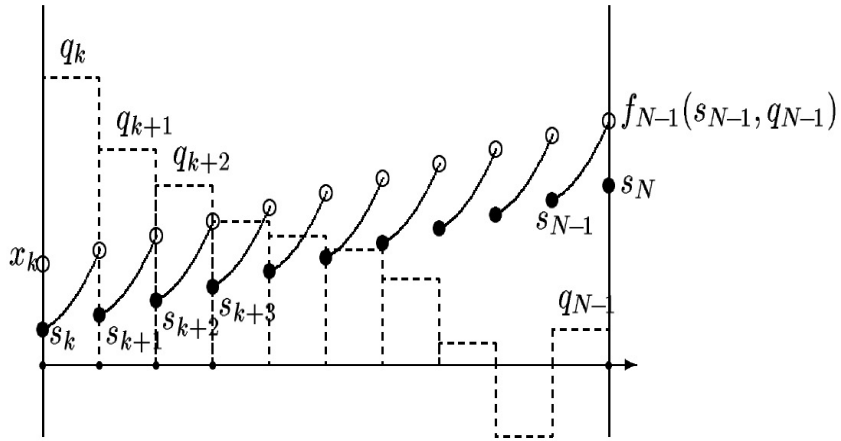
$$x_k - s_k = 0$$

$$f(s_i, u_i) - s_{i+1} = 0$$

The multiple shooting approach [Diehl et al. 2005]

Karush-Kuhn-Tucker optimality condition :

$$\nabla_y \mathcal{L}^k(y) = 0$$



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At each instant

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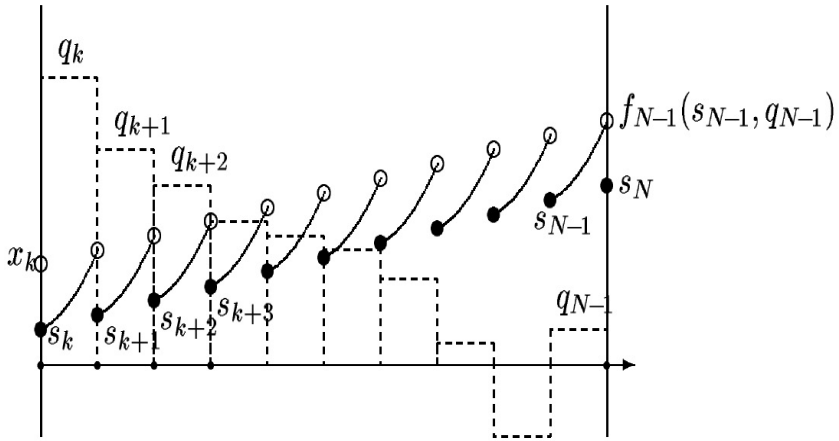

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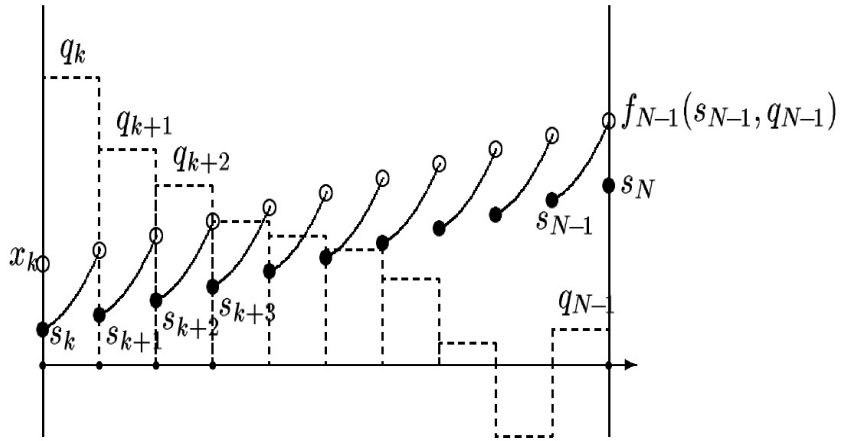

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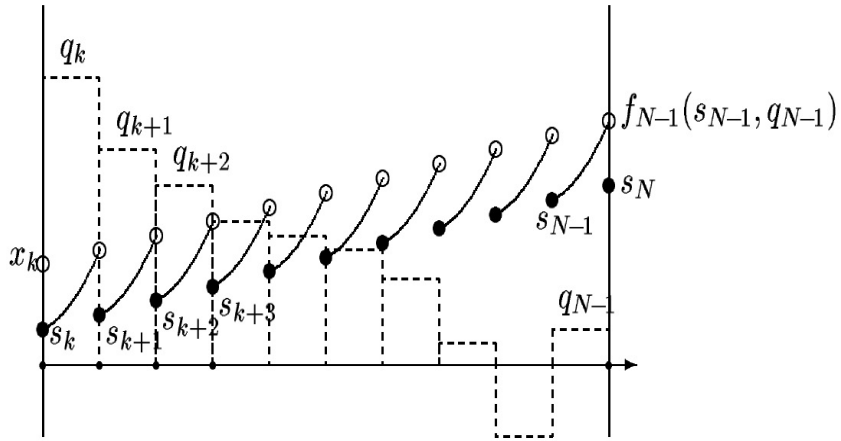

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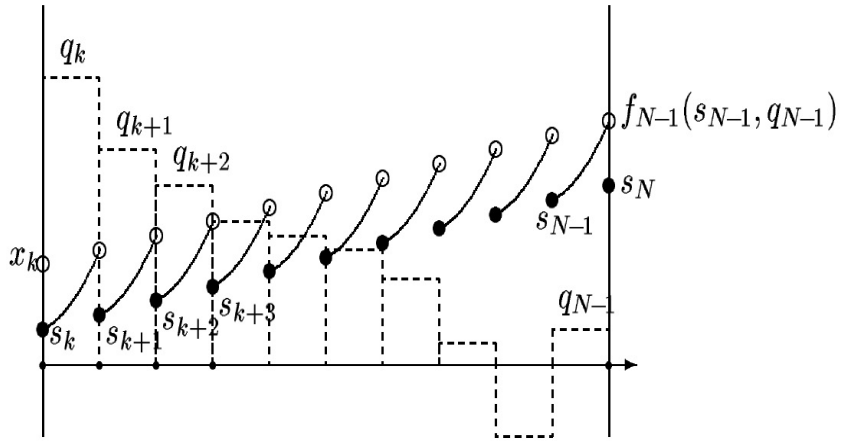

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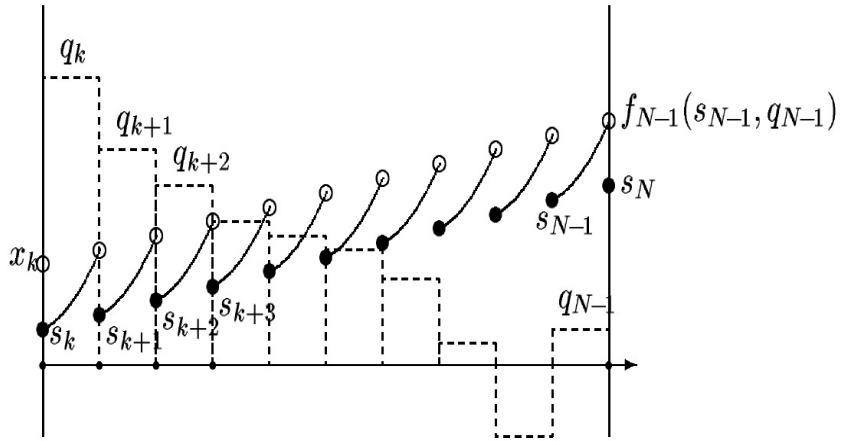
Available software MUSCODII



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The continuation approach

[Ohtsuka et al. 2004]

Hamiltonian system

$$\dot{x} = f(x, u)$$
$$\dot{\lambda} = -H_x(x, u, \lambda, \mu)$$

Hamiltonian

$$H(x, \lambda, \mu, u) =$$
$$L(x, u) + \lambda^T f(x, u) + \mu^T C(x, u)$$

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- Need initialization
- Need differentiability

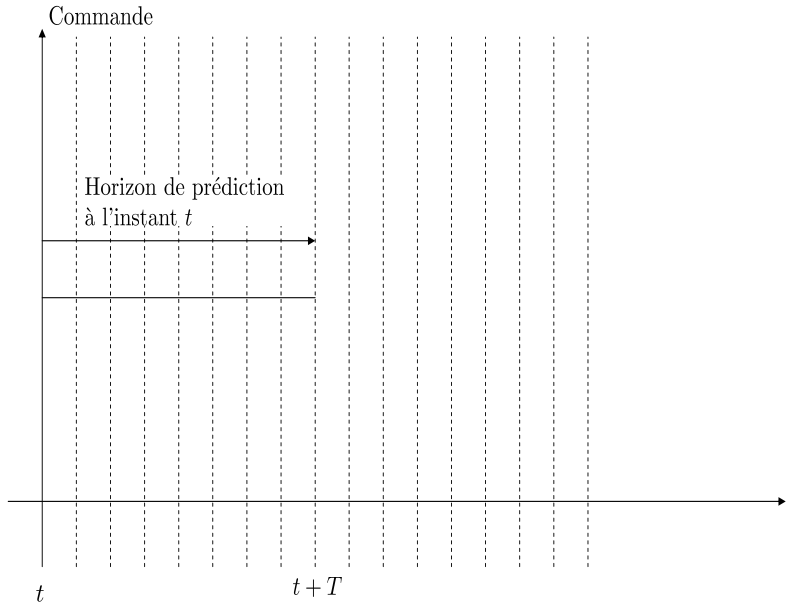
The parametric approach

Fact 1

Simple OL param.

↓

Complex CL behavior



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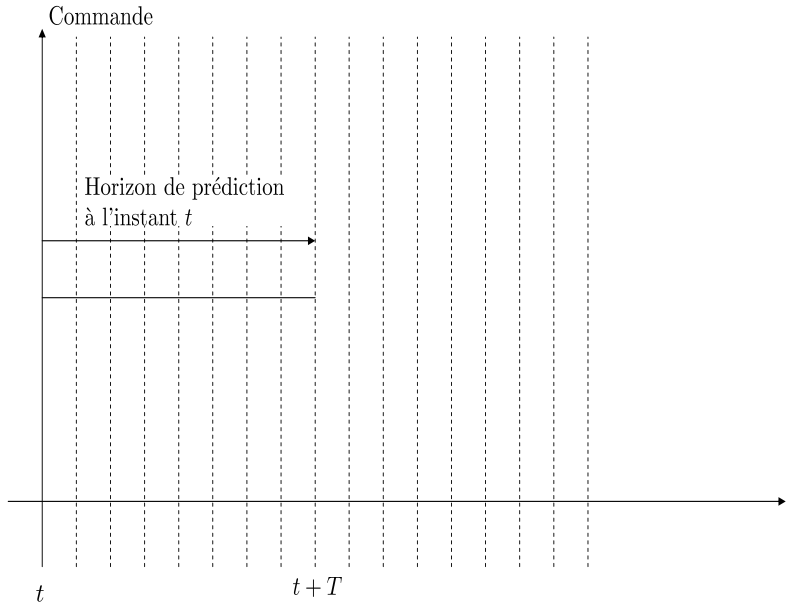

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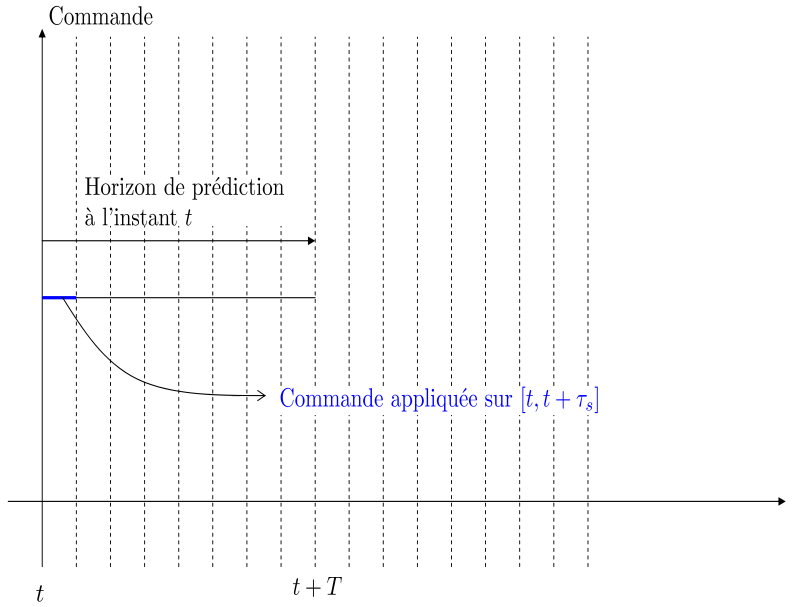

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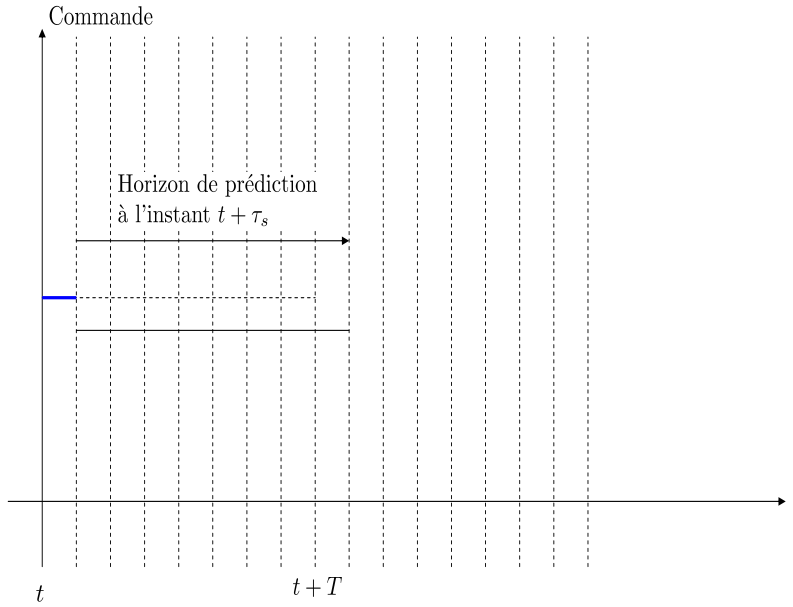

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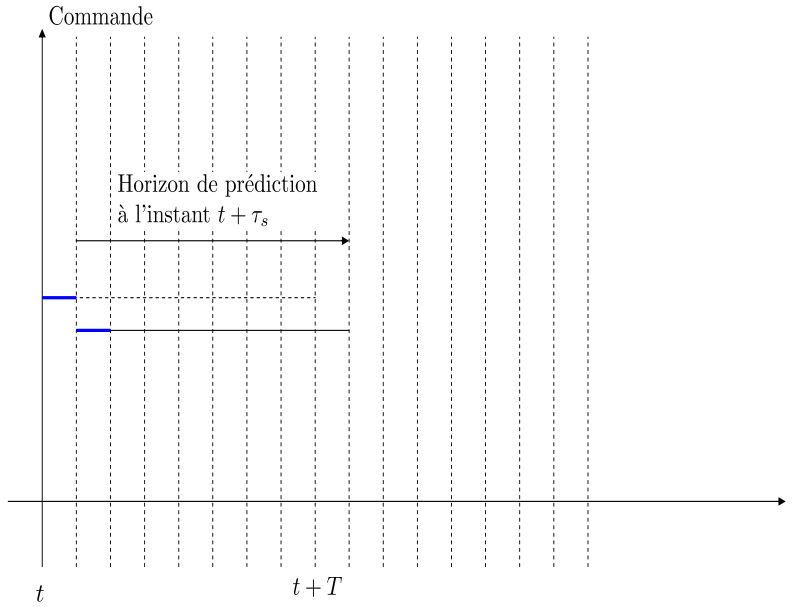

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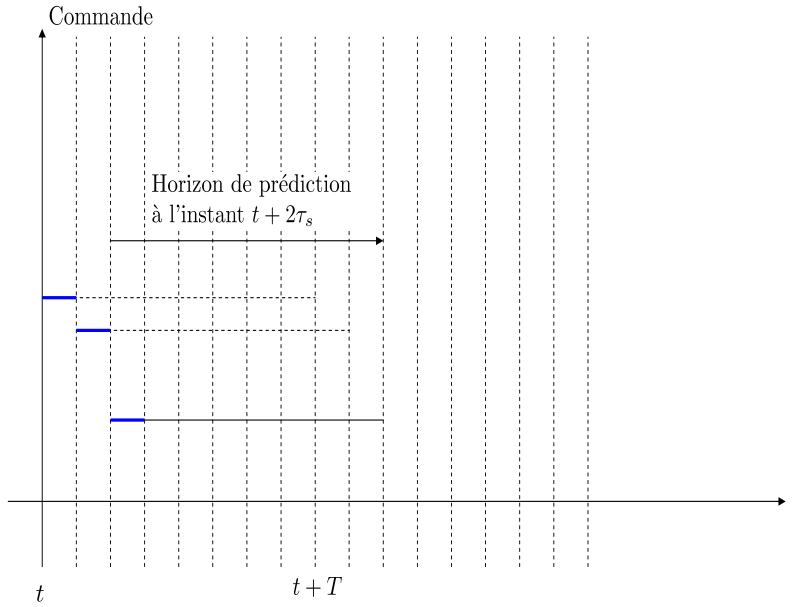

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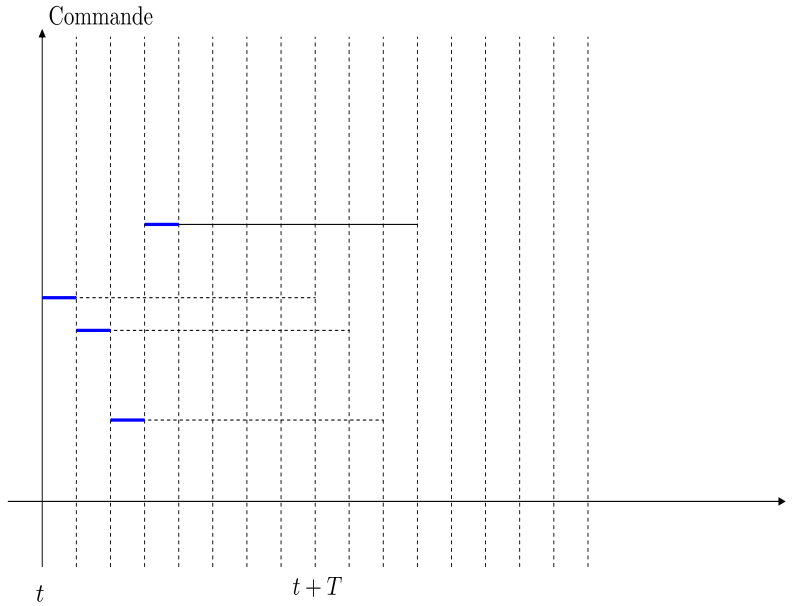

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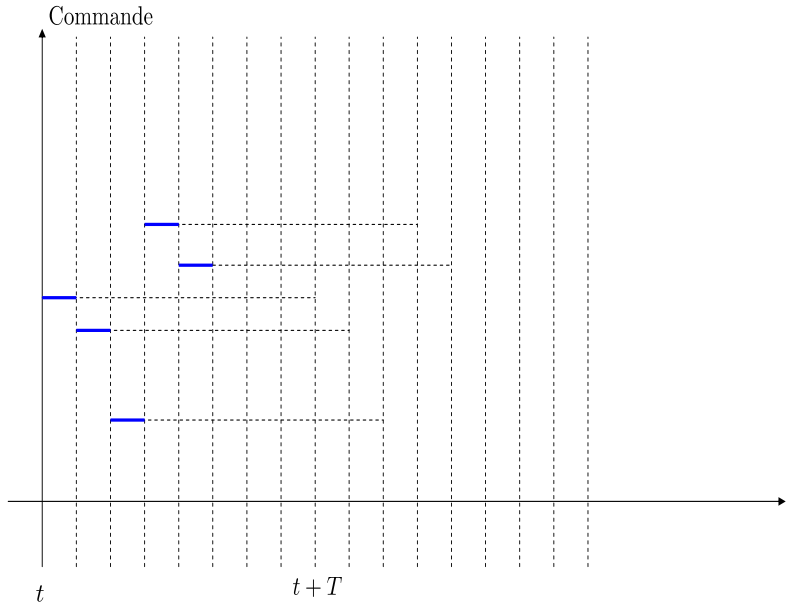

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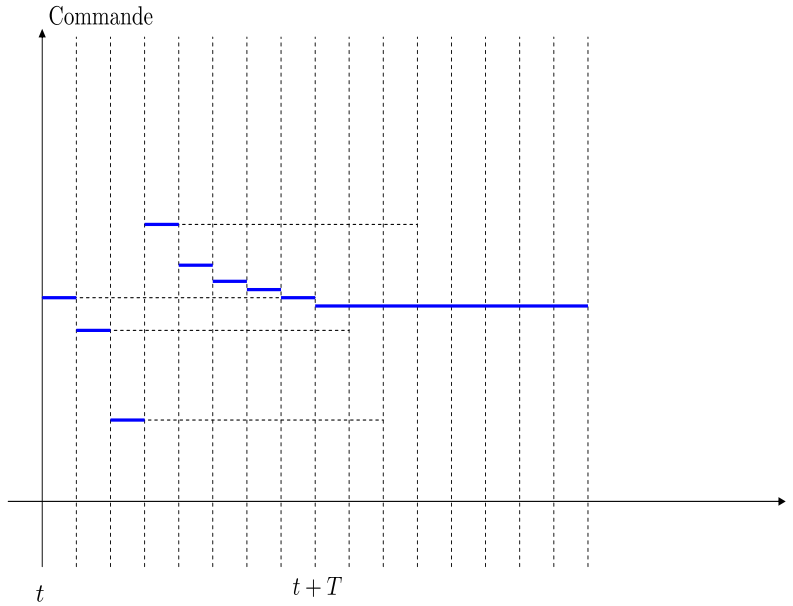

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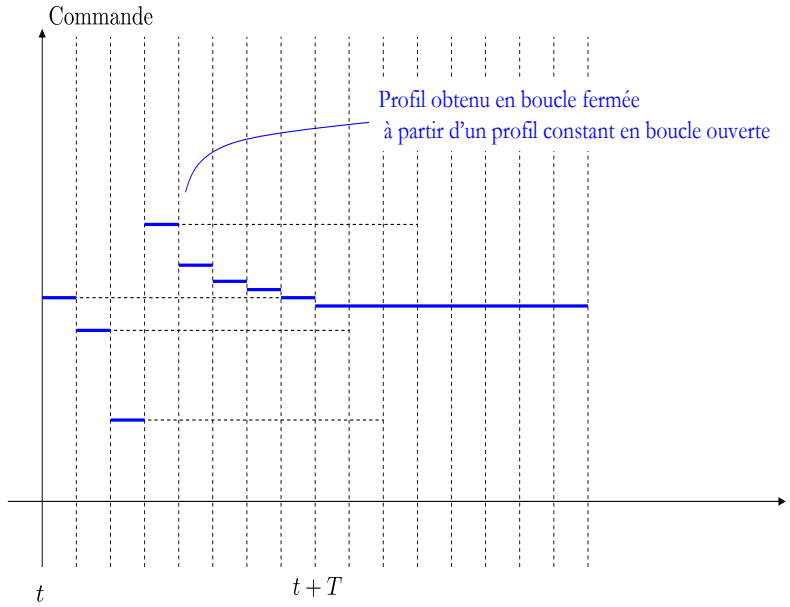


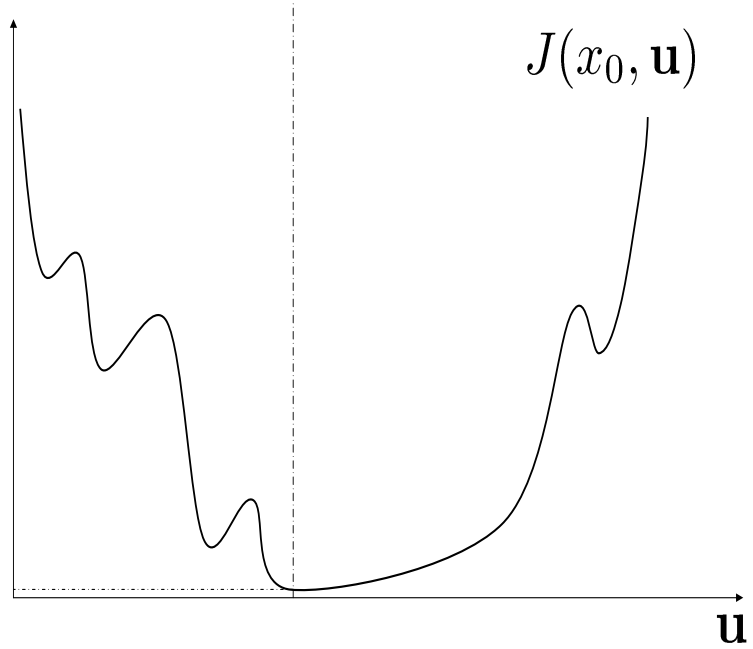
The parametric approach

Fact 2

Solve exactly a sub-optimal formulation

could be better than

Solve loosely the exact problem



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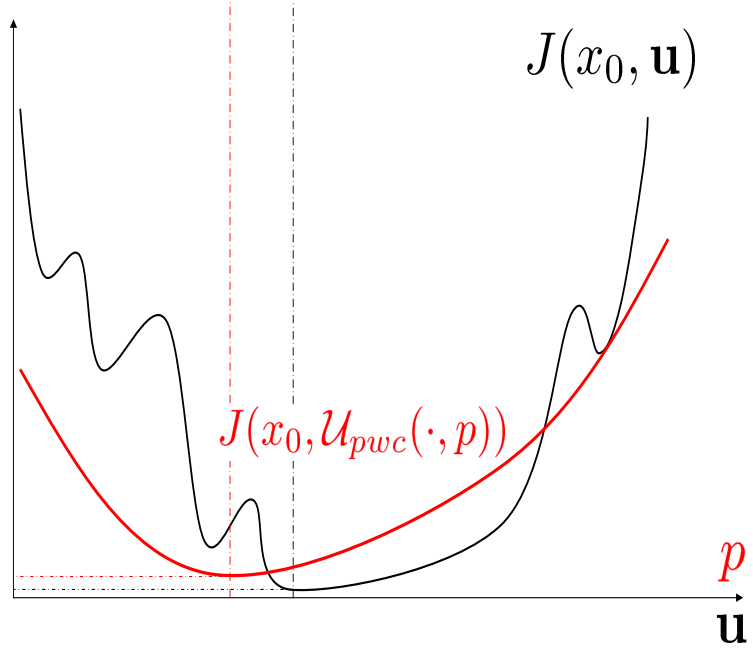

The parametric approach

Fact 3

Constraints can often be embedded in the control parametrization

↓

Optimization improves performance under guaranteed stability and constraints fulfilment

The parametric approach

Fact 4

Reduced dimensional and well posed optimization problems

Render efficient

A family of simple, non smooth and memoryless algorithms that would be out of scope for large scale problems

Problem	n_p	CPU (ms)
Min Intercep. Time	1	2.0
Chained Syst. Stab	1	1.0
PVTOL Stab.	2	1.0
Twin Pendulum	1	5.0
Failure Satellite	2	10.0
AMT Hybrid Veh.	1	1.0
Diesel Air Path	2	10.0

The parametric approach

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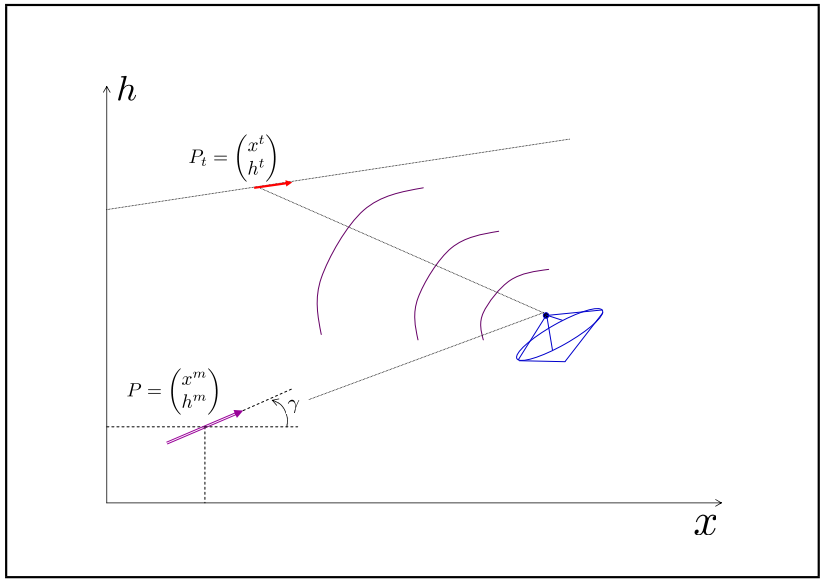
[Alamir, M. Stabilization of Nonlinear Systems Using Receding-Horizon Control Schemes : A Parametrized Approach for Fast Systems. Lecture Notes in Control and Information Sciences, Springer, London, ISBN 1-84628-470-8 (2006)]

The missile/target interception problem



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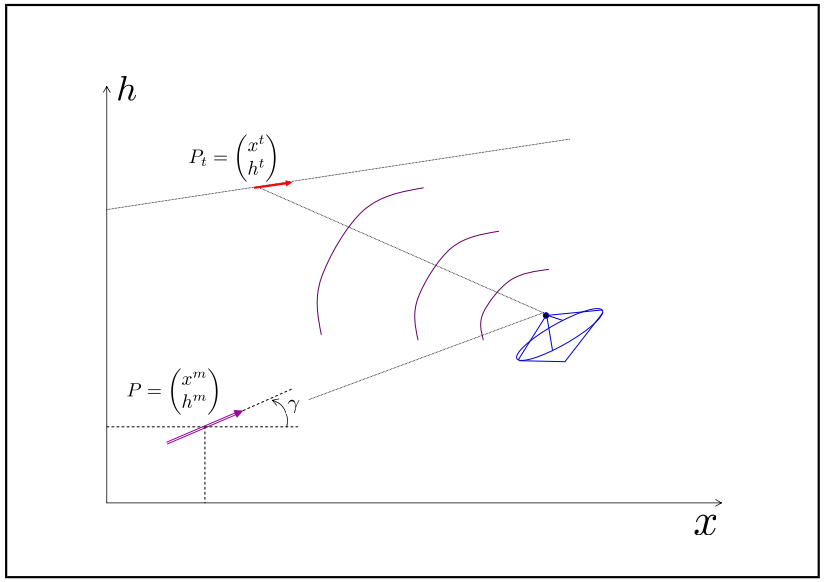

Missile equations

$$
\begin{aligned}
\dot{v} &= (T\cos\alpha - D(h,\alpha))/m - g\sin\alpha \\
\dot{\gamma} &= (L(h,\alpha) + T\sin\alpha)/(mv) - \frac{g}{v}\cos\gamma \\
\dot{x} &= v\cos\gamma \\
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\end{aligned}
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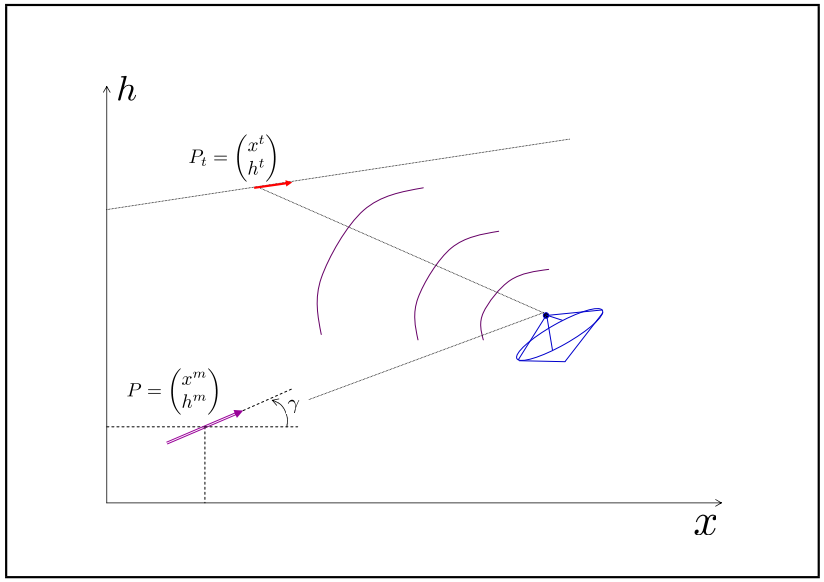
The missile/target interception problem



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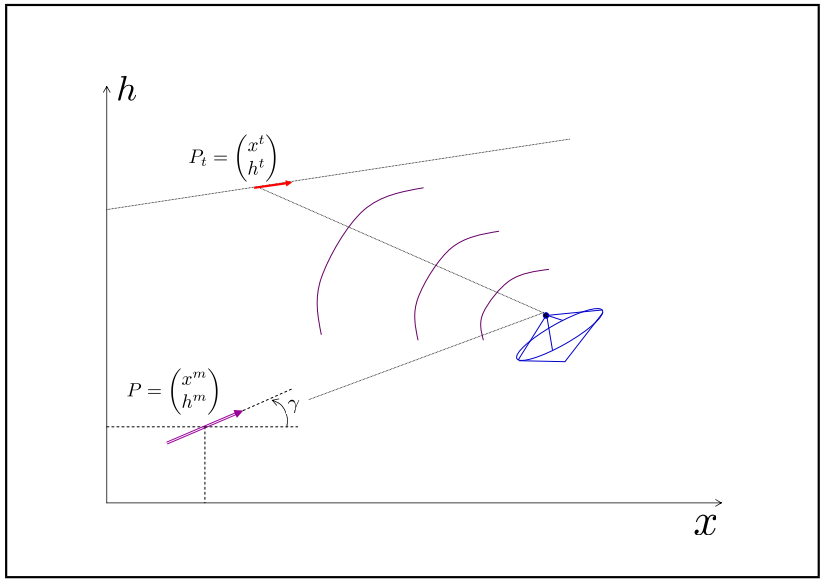

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$$

$M(v,h)$	C_{L_α}	C_{D_0}	k
0.2	9.31	0.242	0.110
0.8	7.65	0.211	0.135
0.93	7.73	0.255	0.134
1.05	9.74	0.406	0.108
1.3	10.03	0.444	0.108
1.6	9.28	0.370	0.116
2.4	9.02	0.254	0.120
3.5	8.02	0.190	0.134
5.0	7.16	0.149	0.153

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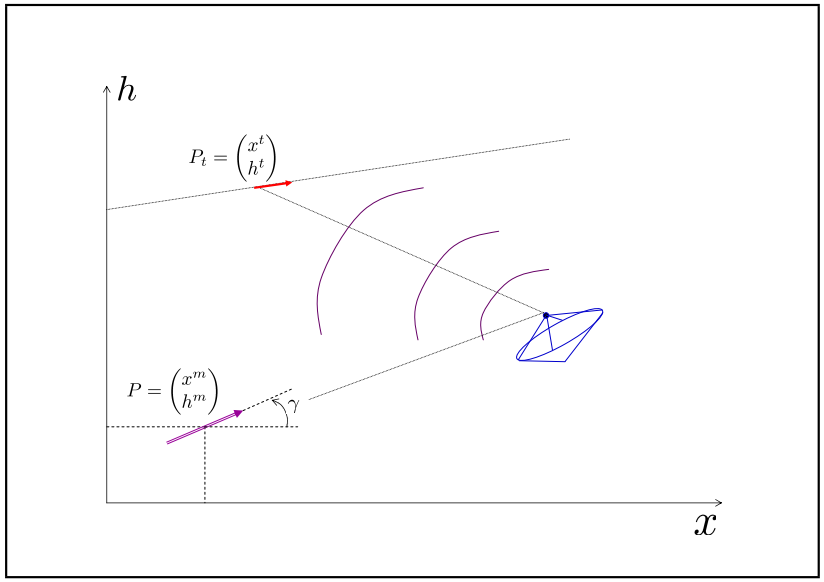

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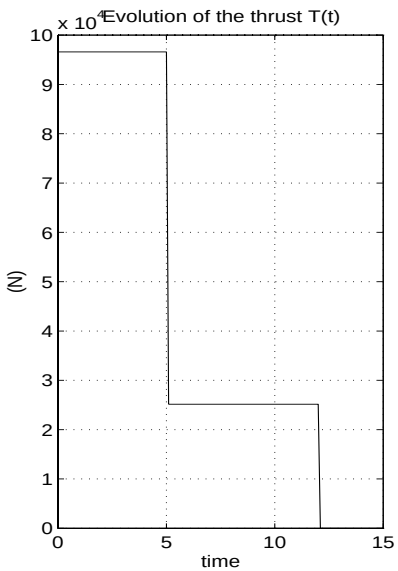
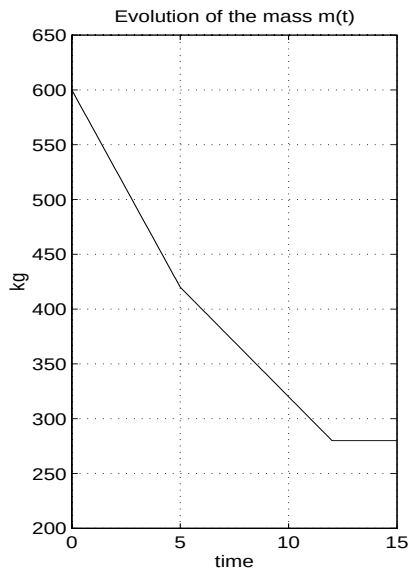
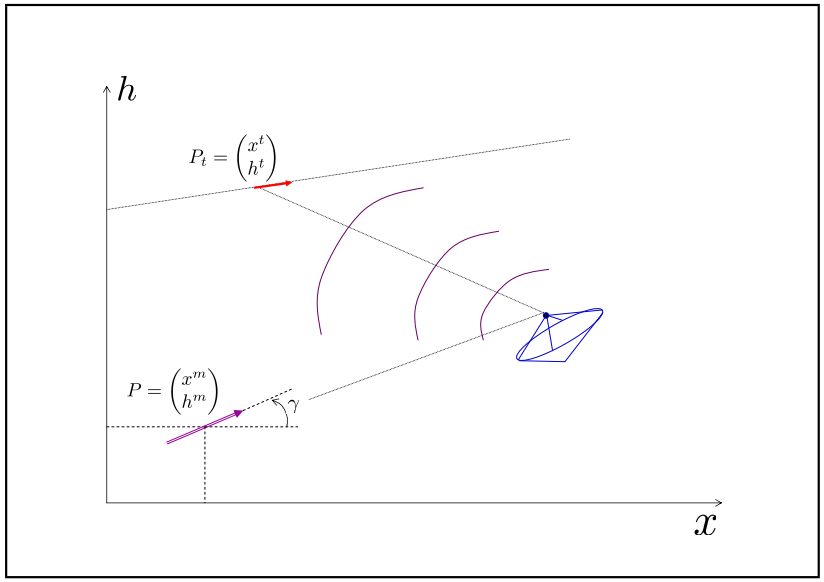




The missile/target interception problem



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$$

- Highly nonlinear
- Constrained
- free final time optimal control problem
- Model uncertainties
- Uncertainties on target's movement

NMPC with scalar control parametrization

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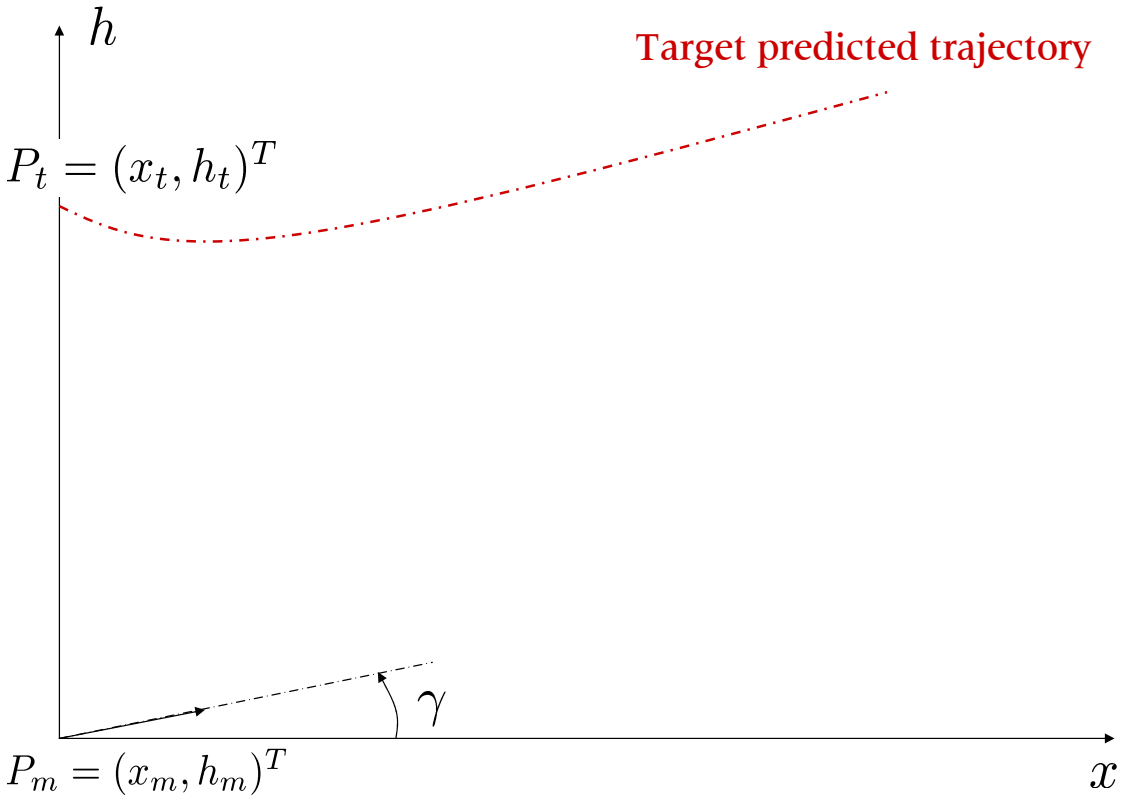
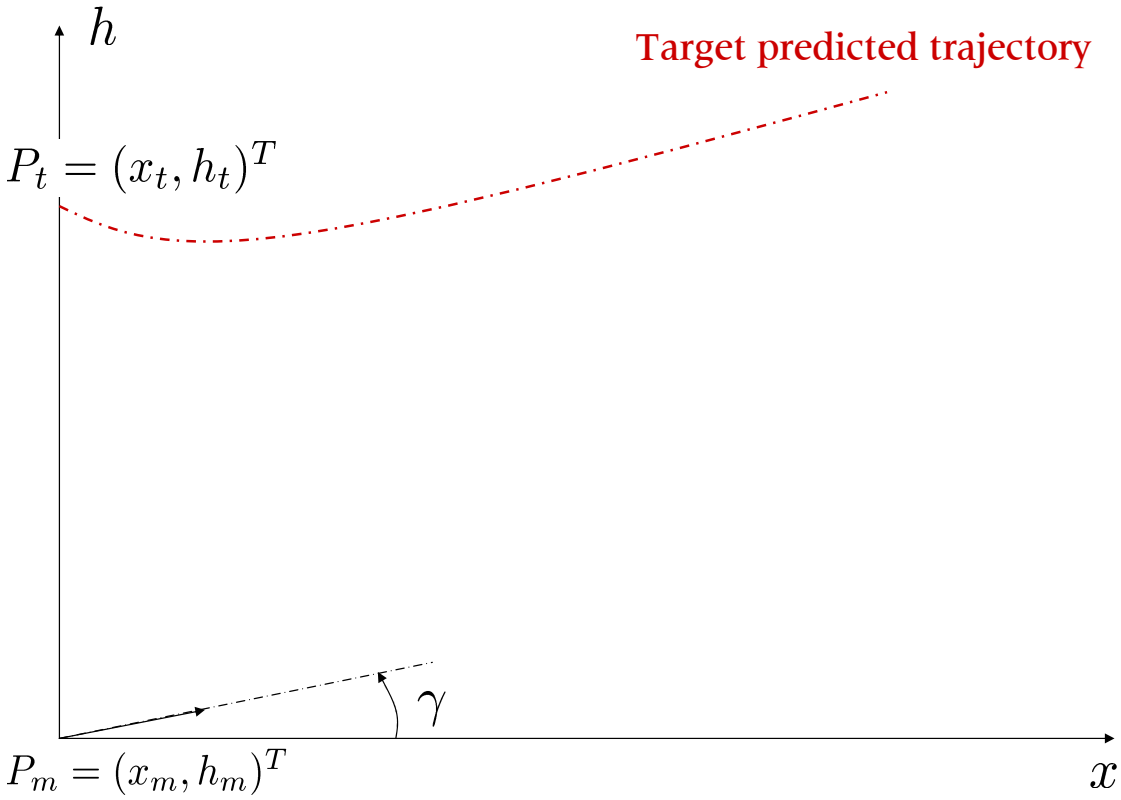
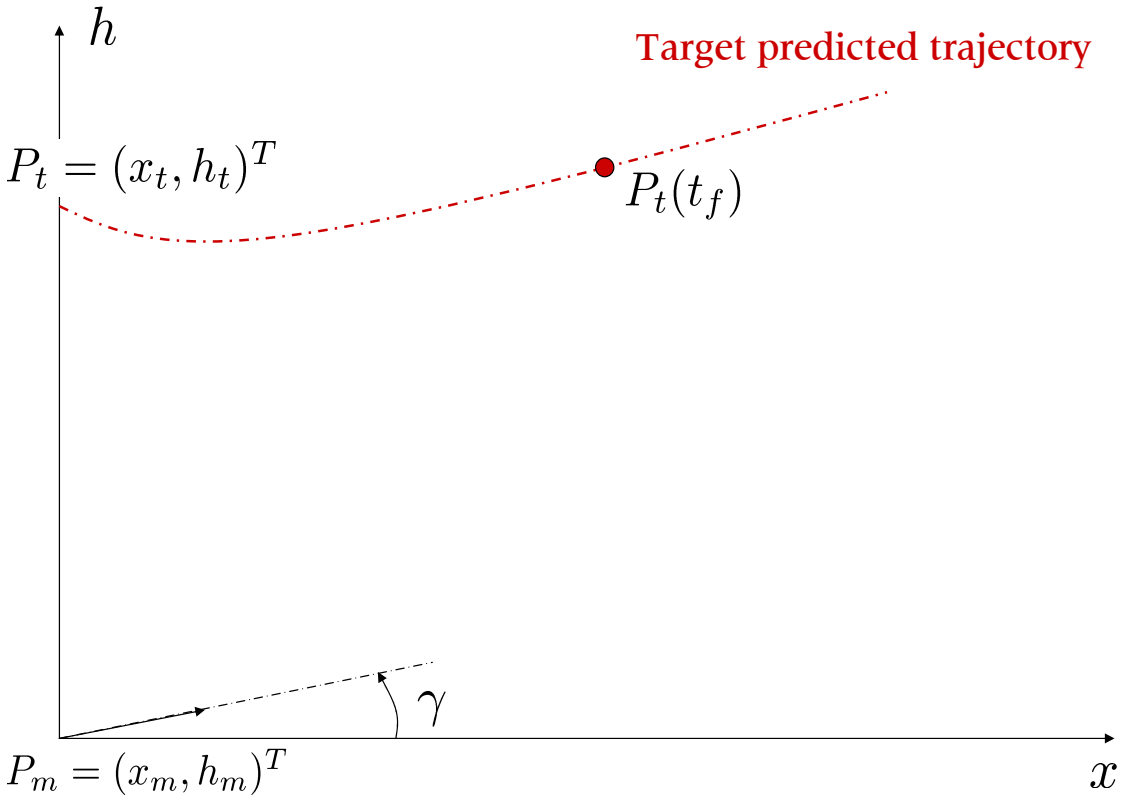

h
Target predicted trajectory
$P_t = (x_t, h_t)^T$
γ
$P_m = (x_m, h_m)^T$
x



Assume that the interception time is $t_f > 0$



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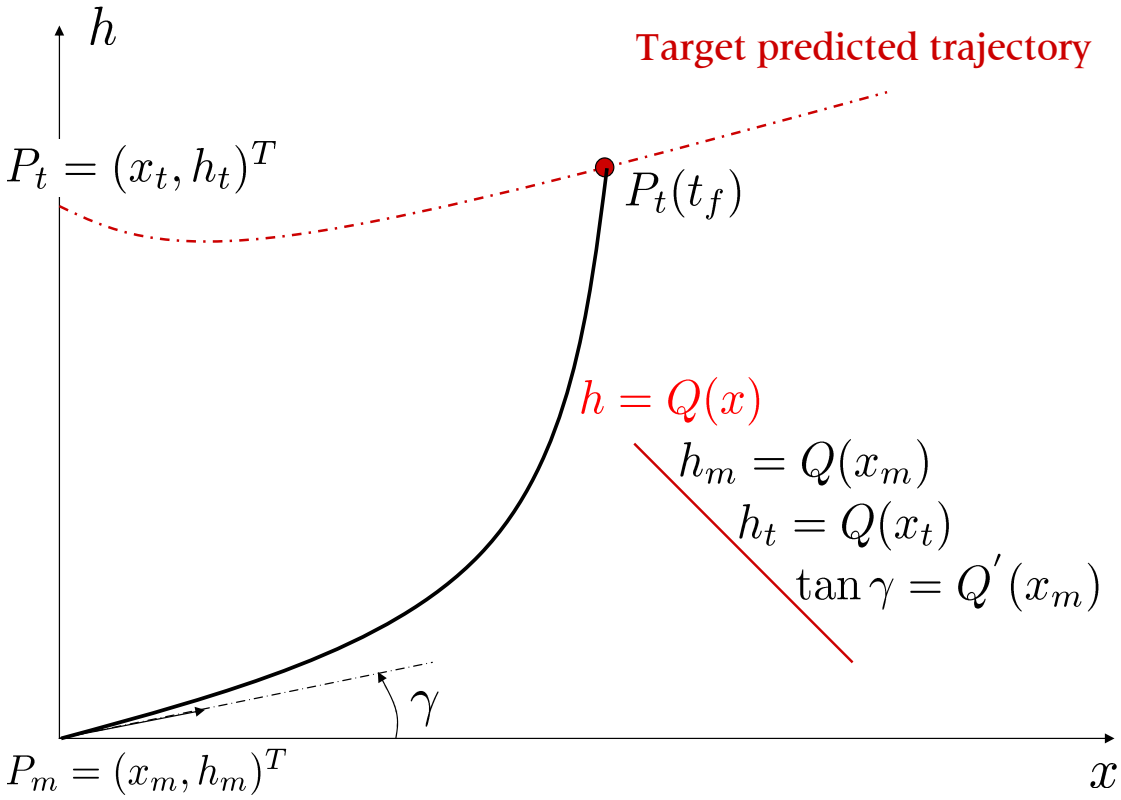
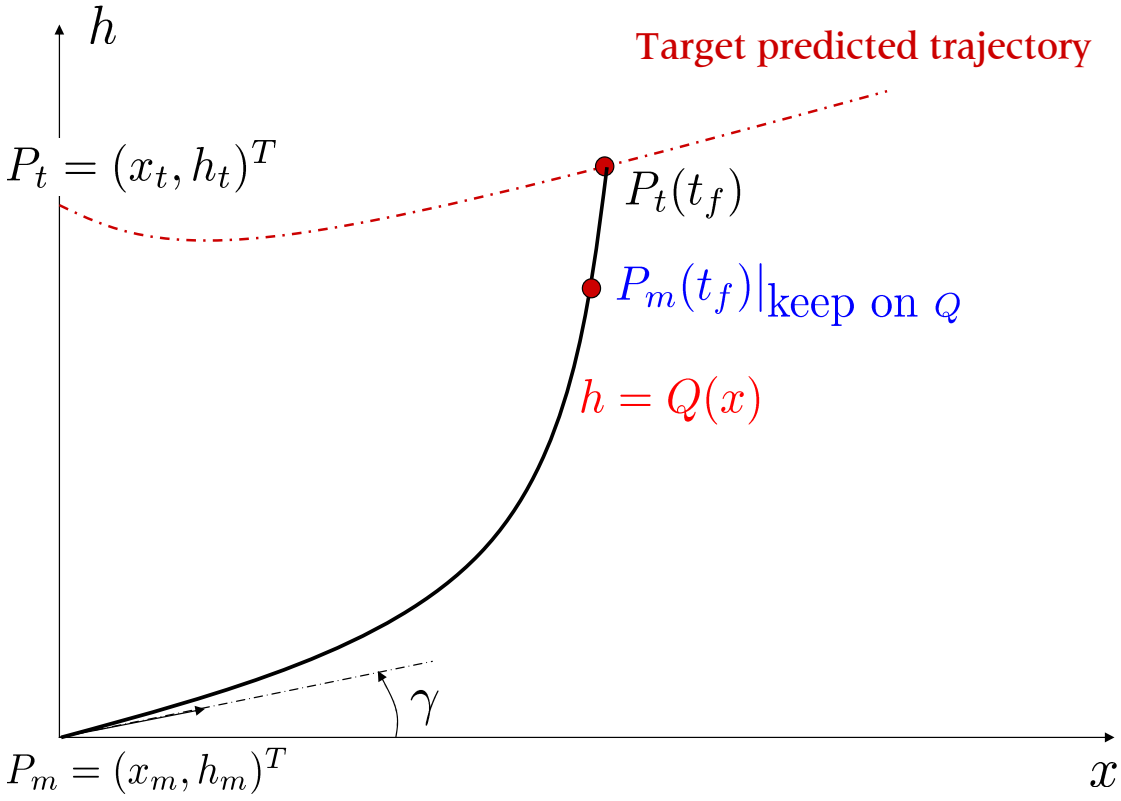


Assume that the interception time is $t_f > 0$

Consider the Parabola $Q(x)$ joining P_m to $P_t(t_f)$



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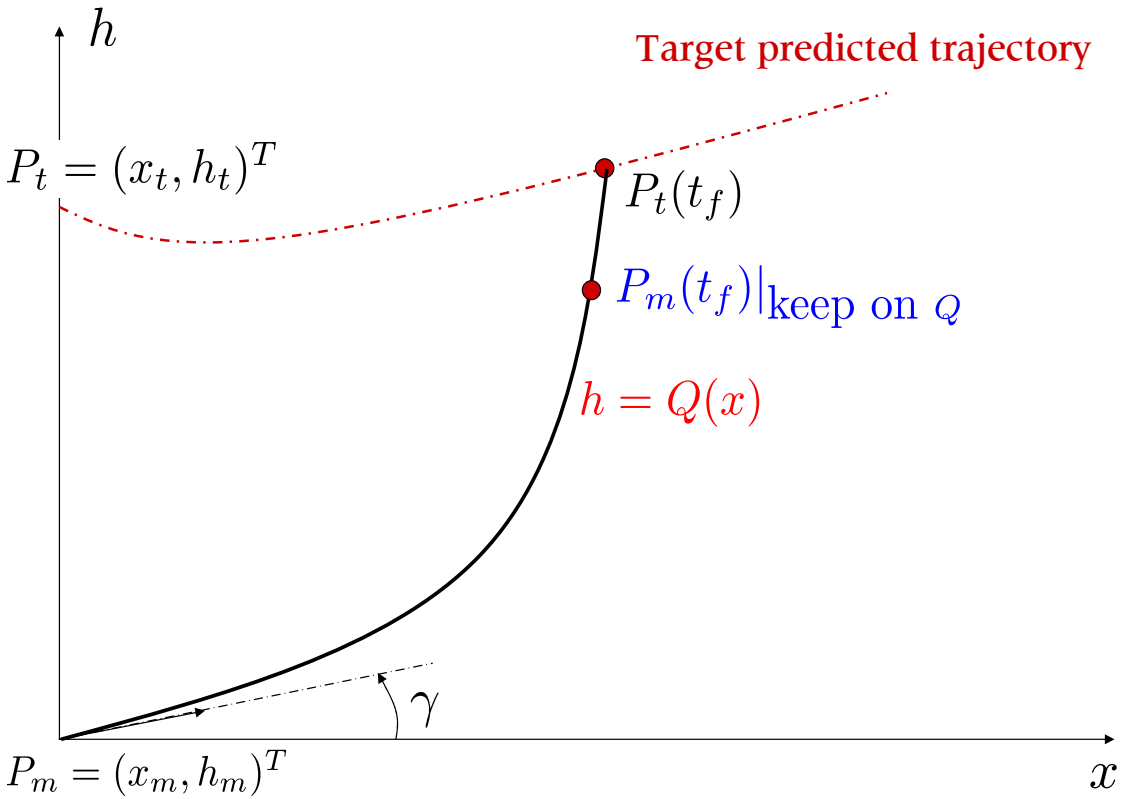
For arbitrary t_f, $P_m(t_f)|_{\text{keep on } Q} \neq P_t(t_f)$



The on-line optimization problem for RHC at decision instant t_k is

$$\min_{t_f} \quad \Big[\beta(t_f, X(t_k))\Big] = \Big\| P_m(t_f, X(t_k))_{|\text{keep on } Q} - P_t(t_f) \Big\|$$

$$\min_{t_f} \quad \left[\beta(t_f, X(t_k))\right] = \left\| P_m(t_f, X(t_k))_{|\text{keep on } Q} - P_t(t_f) \right\|$$

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Feedback 1

Solve exactly

$$\beta(t_f, X(t_k)) = 0$$

by dichotomy

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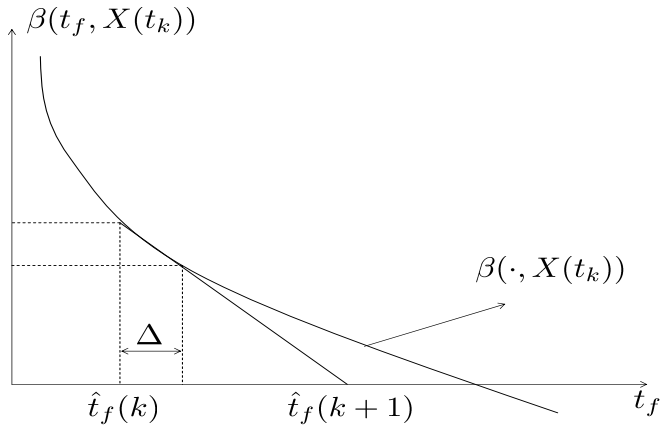
Feedback 2

Real-Time gradient iteration



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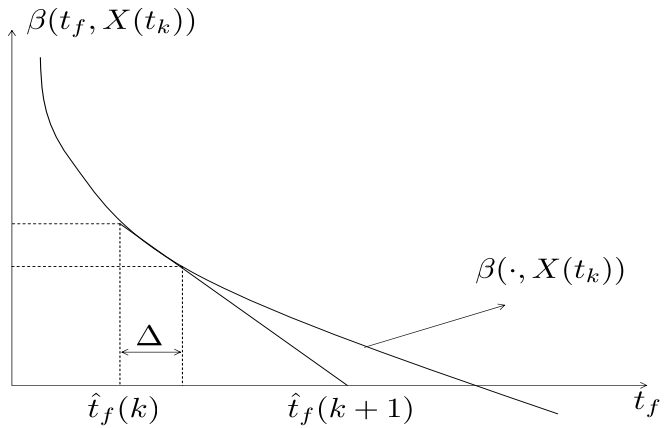
Feedback 1	Feedback2
18 ms	1 ms

Execution time on PC-Pentium II 1.3GHz

Feedback 2

Real-Time gradient iteration



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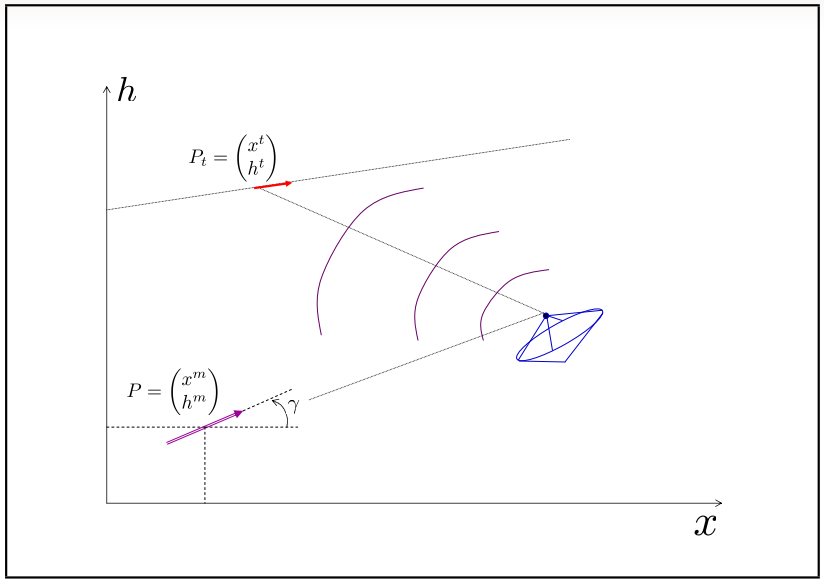

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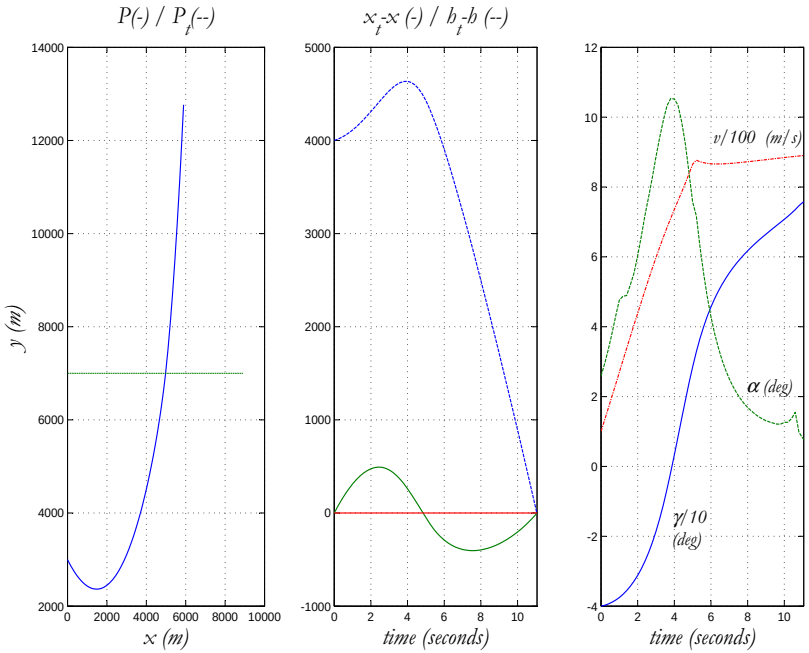
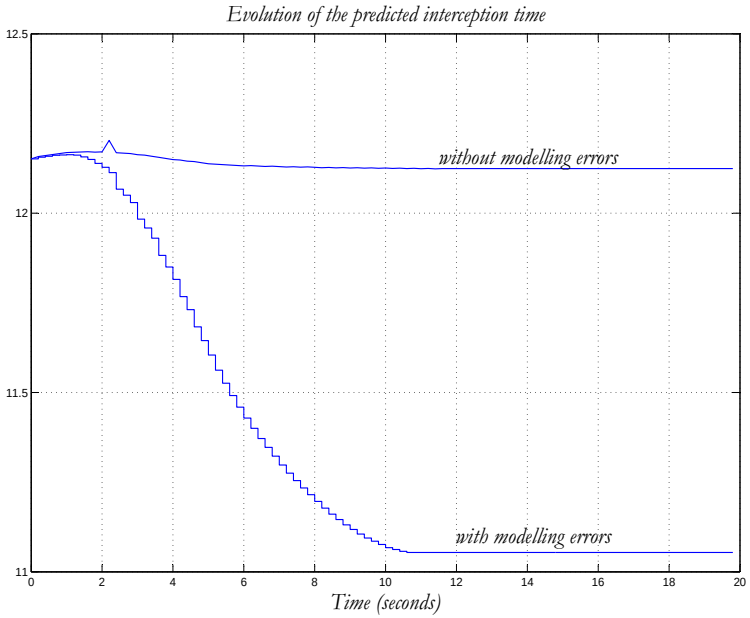
$$

Description of uncertainties

$$
\begin{aligned}
C'_{D_0} &:= \lambda_1 C_{D_0} \\
k' &= \lambda_2 k \\
C'_{L_\alpha} &= \lambda_3 C_{L_\alpha},
\end{aligned}
$$

Simulations

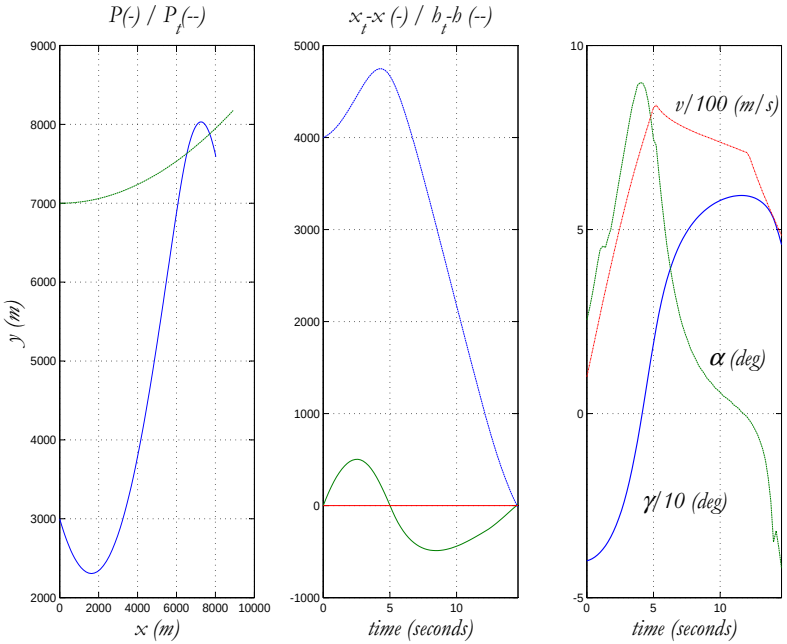


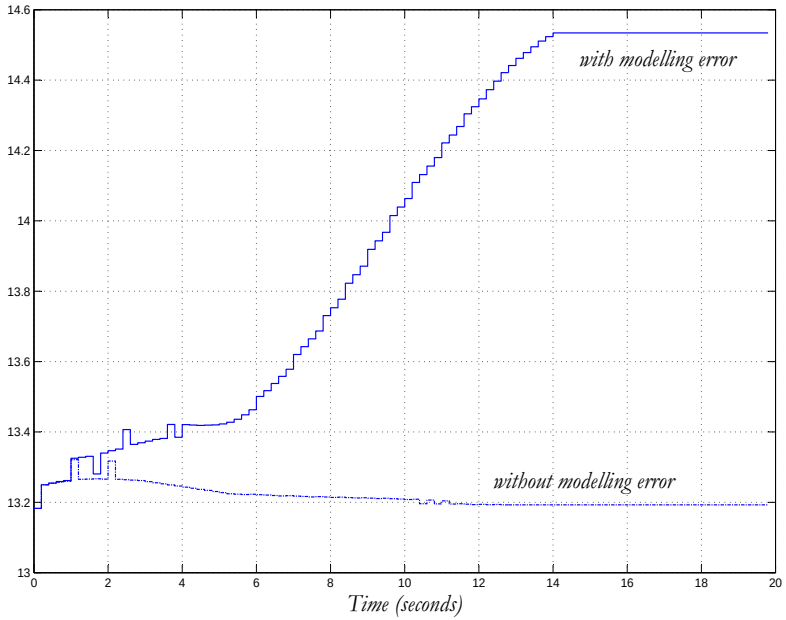


Closed-loop behavior under Feedback 1 and uncertainties given by

$$\lambda_1 = 0.8; \quad \lambda_2 = 0.6; \quad \lambda_3 = 1.25$$

Simulations





Closed-loop behavior under Feedback 1 and uncertainties given by

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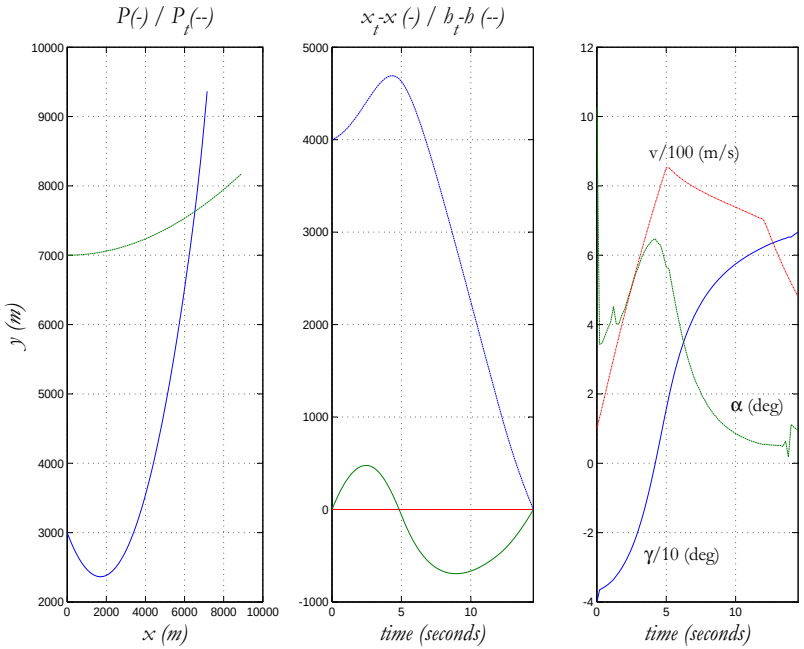
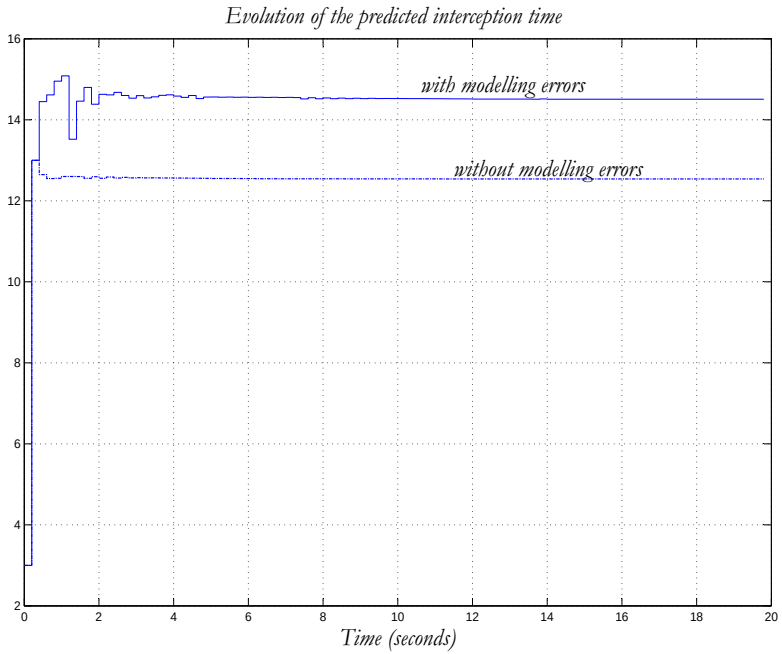
Simulations





Closed-loop behavior under Feedback 2 and uncertainties given by

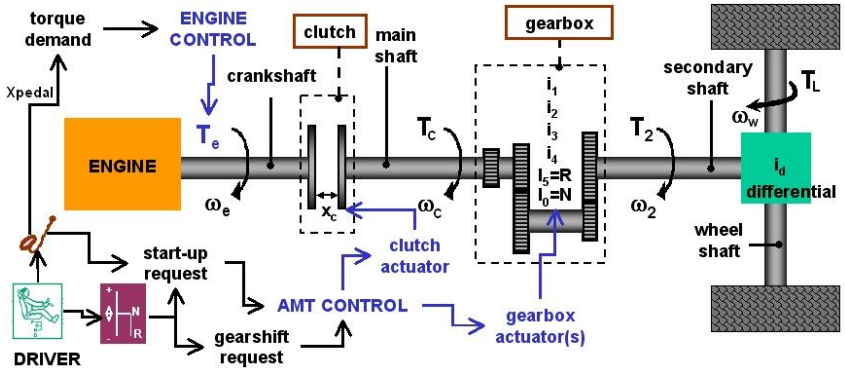
$$\lambda_1 = 1.2; \quad \lambda_2 = 0.6; \quad \lambda_3 = 1.25$$

[see Alamir, M. *Nonlinear Receding Horizon sub-optimal Guidance Law for minimum interception time problem*, Control Engineering Practice 9, No.1, 107-116 (2001)]

The Automated Manual Transmission

Trade-off between

- Manual : efficient, low weight, low cost
- Automated : better comfort / high cost & consumption



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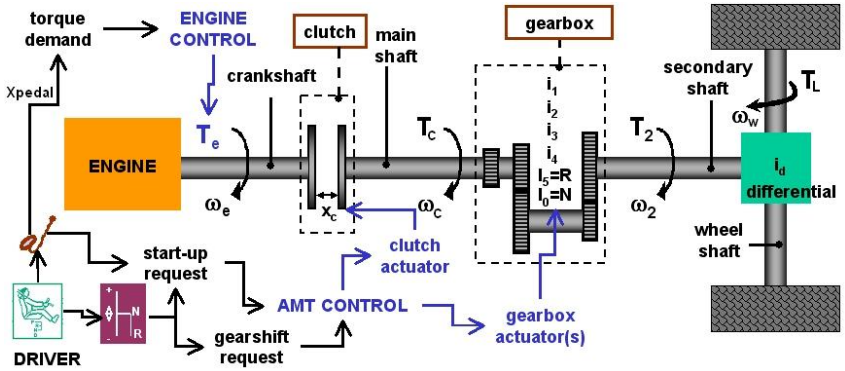

$$J_e\dot{\omega}_e = T_e - sign(\omega_{sl}) \cdot T_c(x_c)$$

$$[J_c + J_{eq}(i_g, i_d)]\dot{\omega}_c = sign(\omega_{sl}) \cdot T_c(x_c) - \frac{1}{i_g i_d}\left[k_{tw}\theta_{cw} + \beta_{tw}\left(\frac{\omega_c}{i_g i_d} - \omega_w\right)\right]$$

$$J_w\dot{\omega}_w = k_{tw}\theta_{cw} + \beta_{tw}\left(\frac{\omega_c}{i_g i_d} - \omega_w\right) - T_L(\omega_w)$$

$$\dot{\theta}_{cw} = \frac{\omega_c}{i_g i_d} - \omega_w$$

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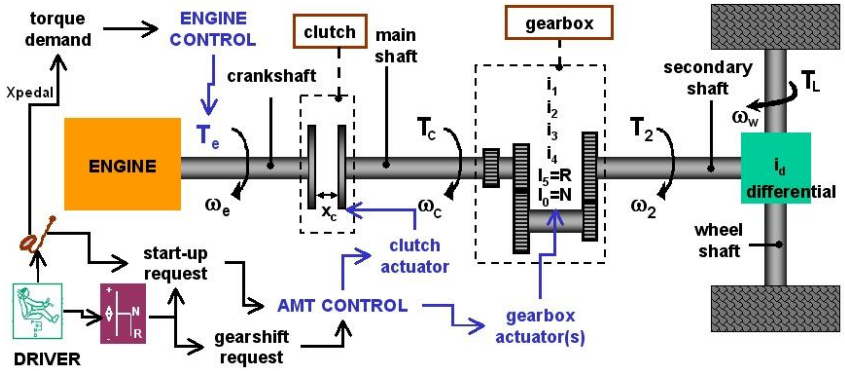

Control objective :

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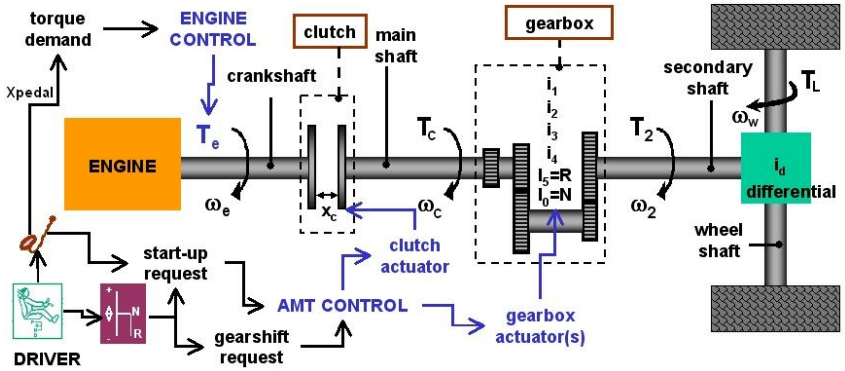

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- Smooth transitions for $\omega_{sl} \longrightarrow 0$

  (this is intimately linked to the dynamic of the slip velocity $\omega_{sl} = \omega_e - \omega_c$)

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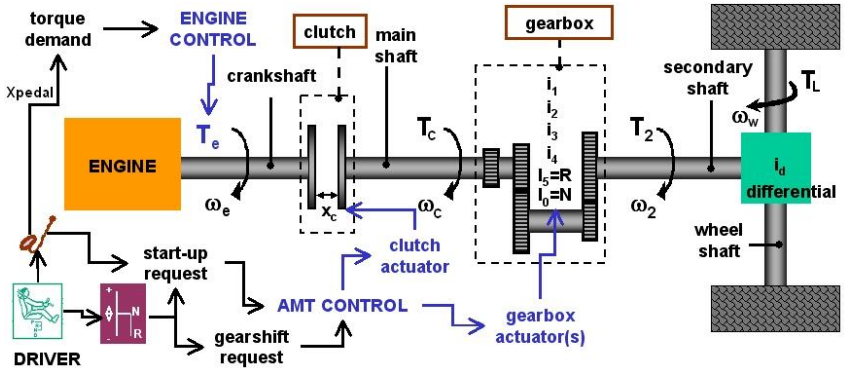

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  The torque must be somehow related to the pedal's position

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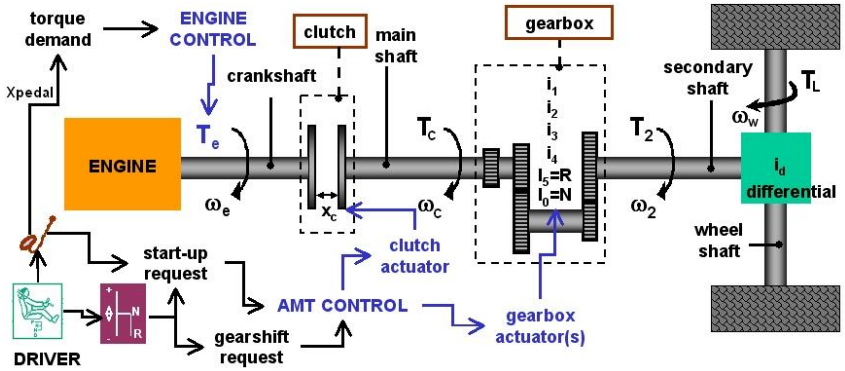

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- Transparency

  The torque must be somehow related to the pedal's position
- Control of the engine velocity :

$$\omega_e^{ref} = max\left\{\omega_e^0, \mathcal{T}^{-1}\left(T_e^d(X_{pedal}, \omega_e)\right)\right\}$$

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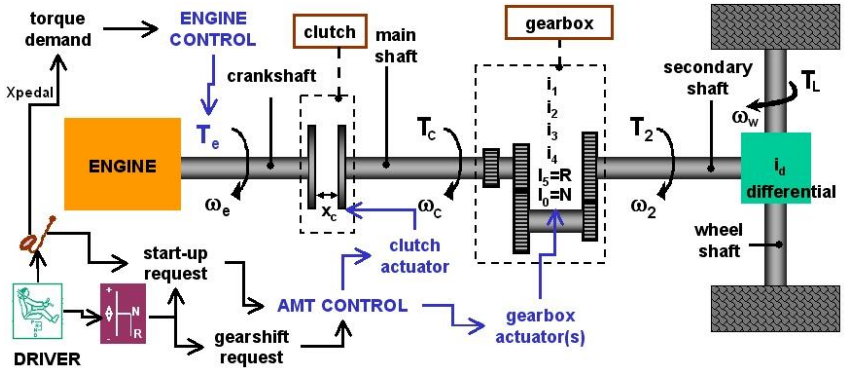

Constraints :

- Torque saturation

$$\begin{aligned} T_e &\in [T_e^{min}, T_e^{max}(\omega_e)] \\ T_c &\in [T_c^{min}, T_c^{max}(\omega_e)] \end{aligned}$$

- Torque variation rate saturation

$$\begin{aligned} \dot{T}_e &\in [\dot{T}_e^{min}, \dot{T}_e^{max}] \\ \dot{T}_c &\in [\dot{T}_c^{min}, \dot{T}_c^{max}] \end{aligned}$$

Some remarks

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- A conventional constrained linear quadratic with the required prediction horizon cannot be solved within the allowable sampling period (50 ms).
- Existing works suggested a first time period of open-loop control.
- PWA related approaches are still not real-time compatible.
- The MPC stage uses the torque set-points T_e^{sp} and T_c^{sp} as control variables, namely :

$$u := \left(T_e^{sp}, T_c^{sp}\right)^T$$

Some remarks

- Need for a simplified model
- A conventional constrained linear quadratic with the required prediction horizon cannot be solved within the allowable sampling period (50 ms).
- Existing works suggested a first time period of open-loop control.
- PWA related approaches are still not real-time compatible.
- The MPC stage uses the torque set-points T_e^{sp} and T_c^{sp} as control variables, namely :

$$u := \left(T_e^{sp}, T_c^{sp}\right)^T$$

- Therefore, the closed-loop has to cope with the torque control imperfections & simplified modeling error

Simplified Model for Control

Simplified Model for Control

$$J_e\dot{\omega}_e = T_e - sign(\omega_{sl}) \cdot T_c(x_c)$$

$$[J_c + J_{eq}(i_g, i_d)]\dot{\omega}_c = sign(\omega_{sl}) \cdot T_c(x_c) - \frac{1}{i_g i_d}\left[k_{tw}\theta_{cw} + \beta_{tw}\left(\frac{\omega_c}{i_g i_d} - \omega_w\right)\right]$$

$$J_w\dot{\omega}_w = k_{tw}\theta_{cw} + \beta_{tw}\left(\frac{\omega_c}{i_g i_d} - \omega_w\right) - T_L(\omega_w)$$

$$\dot{\theta}_{cw} = \frac{\omega_c}{i_g i_d} - \omega_w$$

Simplified Model for Control

$$J_e\dot{\omega}_e = u_1 - sign(\omega_{sl}) \cdot u_2 + \delta_e$$

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- δ_e copes with tracking error on T_e^{sp} and T_c^{sp}.

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- δ_c copes with
  1. tracking error on T_c^{sp}
  2. non explicitly represented dynamics
  3. unknown load T_L

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- Since ω_e and ω_c are measured, observers can be designed for δ_e and δ_c

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- Over time windows where ω_{sl} keeps a constant sign, the system is linear.
- Since ω_e and ω_c are measured, observers can be designed for δ_e and δ_c
- MPC design uses observer generated $\hat{\delta}_e$ and $\hat{\delta}_c$ with constant dynamics.

The Parameterized NMPC scheme

Model for prediction

$$J_e \dot{\omega}_e = u_1 - sign(\omega_{sl}) \cdot u_2 + \hat{\delta}_e$$
$$[J_c + J_{eq}] \dot{\omega}_c = sign(\omega_{sl}) \cdot u_2 - \hat{\delta}_c$$

The Parameterized NMPC scheme

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- Consider a prediction horizon $N_p \cdot \tau_s$

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$$\omega_{sl}^{ref}(k+i, p) = \frac{1 - i/p}{1 + \lambda \cdot i/p} \cdot \omega_{sl}(k)$$

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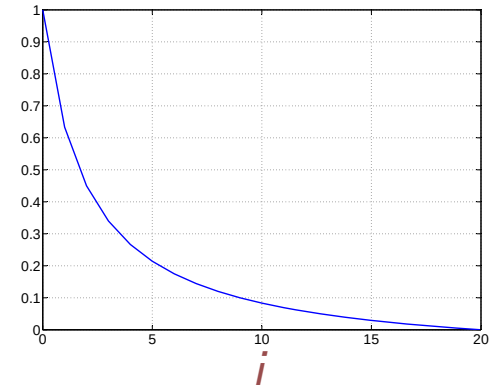
$\frac{1-i/p}{1+\lambda \cdot i/p}$, $N_p = 20$, $p = N_p$



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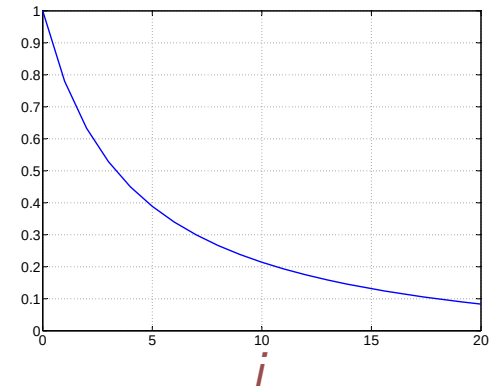
$\frac{1-i/p}{1+\lambda \cdot i/p}$, $N_p = 20$, $p = 2N_p$



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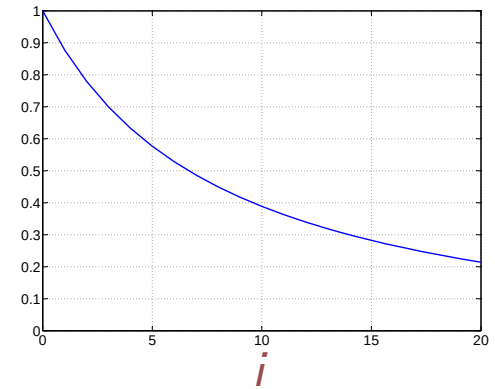
$\frac{1-i/p}{1+\lambda \cdot i/p}$, $N_p = 20$, $p = 3N_p$



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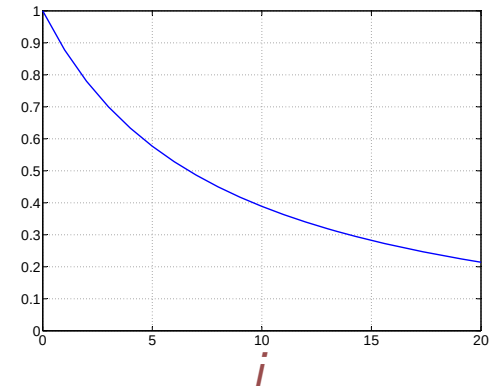

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- p monitors the speed of convergence of ω_{sl} to 0

The Parameterized NMPC scheme

Given X_{pedal}, for each choice of $p \in [N_p, \infty]$, a piecewise constant control profile can be defined according to the following unconstrained problem :

$$\mathcal{U}_{pwc}(p,\omega) := arg\min_{\mathbf{u}} \Omega(\mathbf{u},p,\omega) := \sum_{i=1}^{N_p} \left\| \begin{pmatrix} \omega_{sl}(k+i) - \omega_{sl}^{ref}(k+i,p) \\ \omega_e(k+i) - \omega_e^{ref}(k+i) \end{pmatrix} \right\|_Q^2$$

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Recall that :

$$\begin{aligned} \omega_e^{ref}(k+i) &= max\left\{\omega_e^0, \mathcal{T}^{-1}\left(T_e^d(X_{pedal}, \omega_e(k+i))\right)\right\} \\ \omega_{sl}^{ref}(k+i,p) &= \frac{1 - i/p}{1 + \lambda \cdot i/p} \cdot \omega_{sl}(k) \end{aligned}$$

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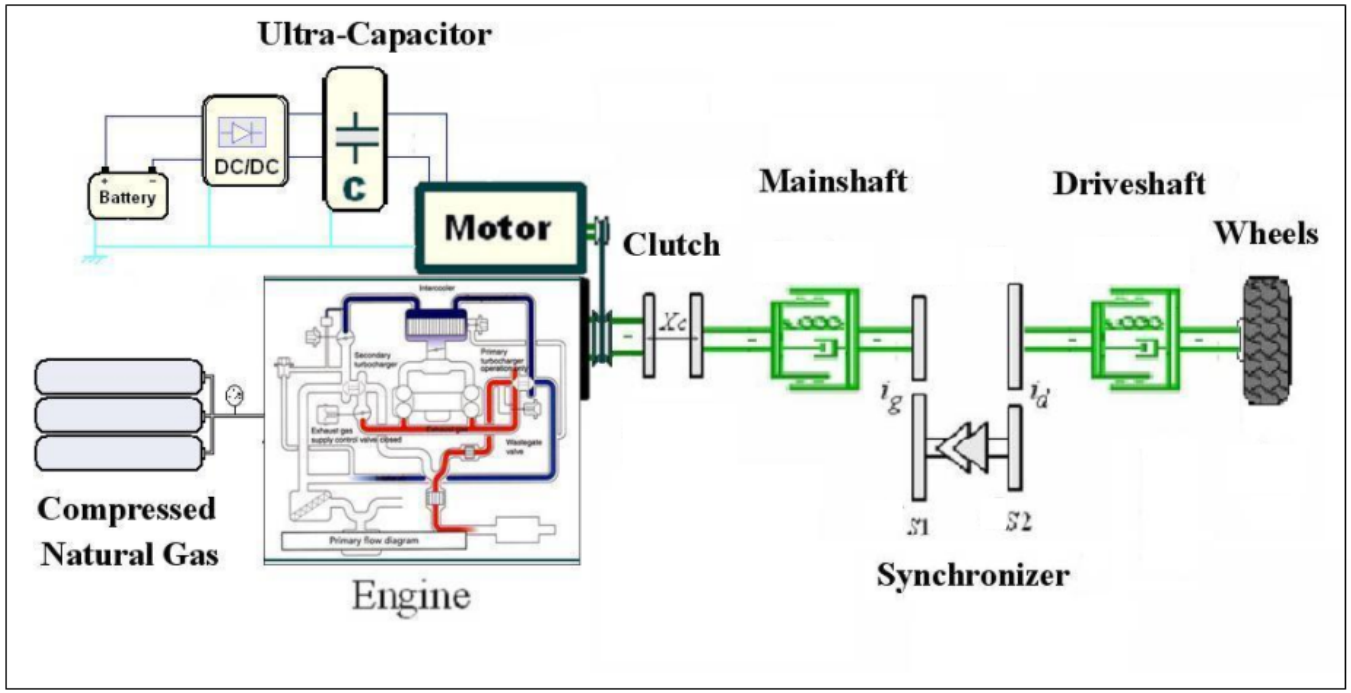
- For each p and X_{pedal}, $\mathcal{U}_{pwc}(p,\omega)$ can be computed analytically
- For sufficiently high p (slow convergence), constraints can be fulfilled

A scalar NMPC with the optimization problem :

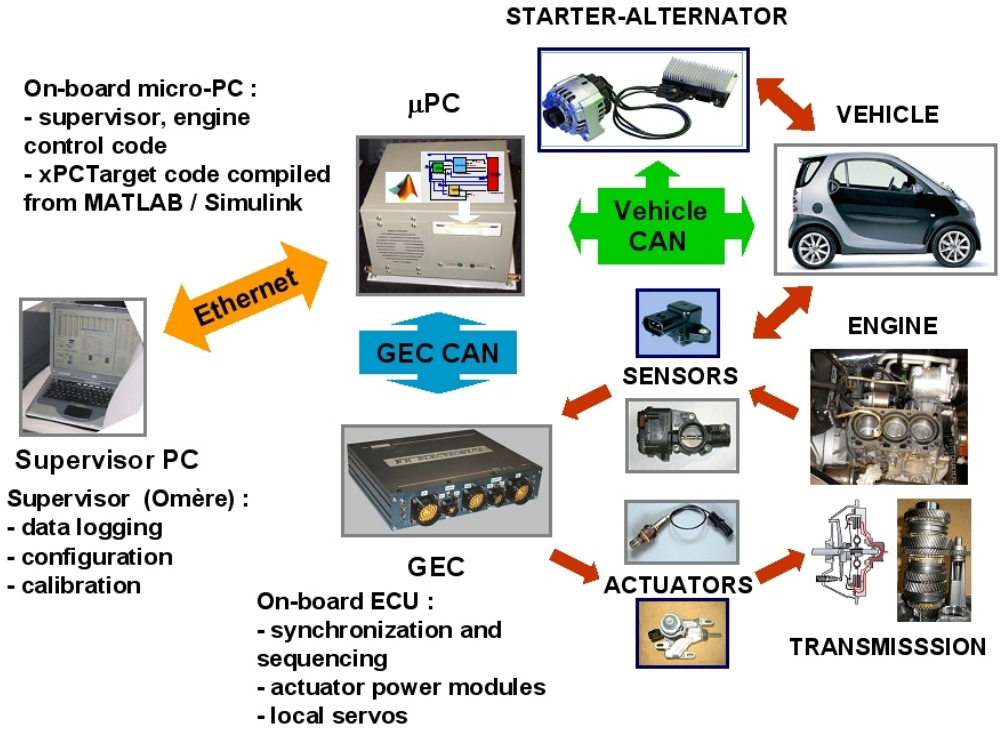
$$\min_{p \in [N_p, N_p^{max}]} J(p,\omega) = |p - t_f(X_{pedal})| \quad \text{under saturation constraints} \quad [C(p,\omega) \leq 0]$$

The IFP's SMART demo architecture (1)



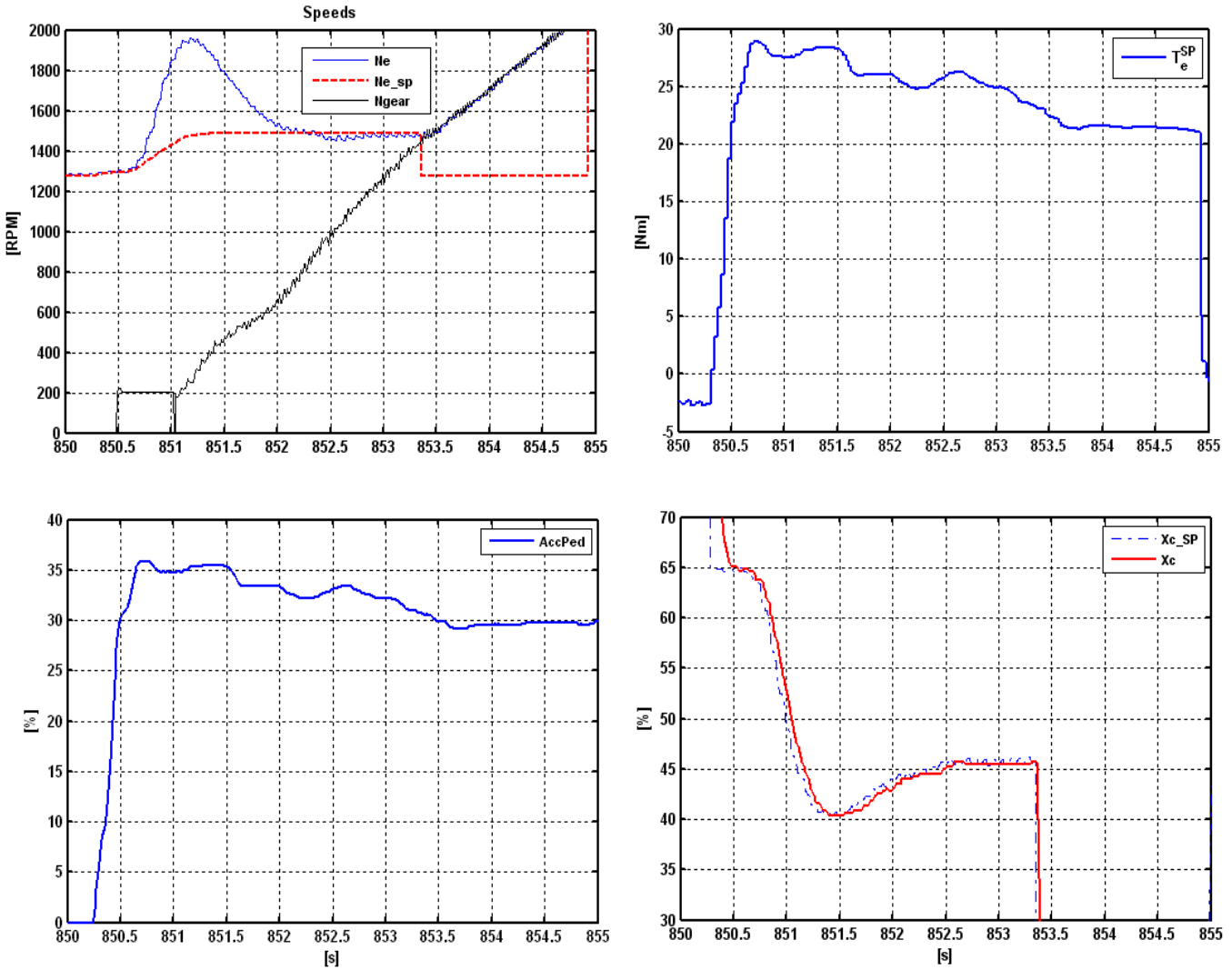
The mild-Hybrid powertrain of the VEHEGAN demo-car

The IFP's SMART demo architecture (2)



VEHEGAN on-board Control System

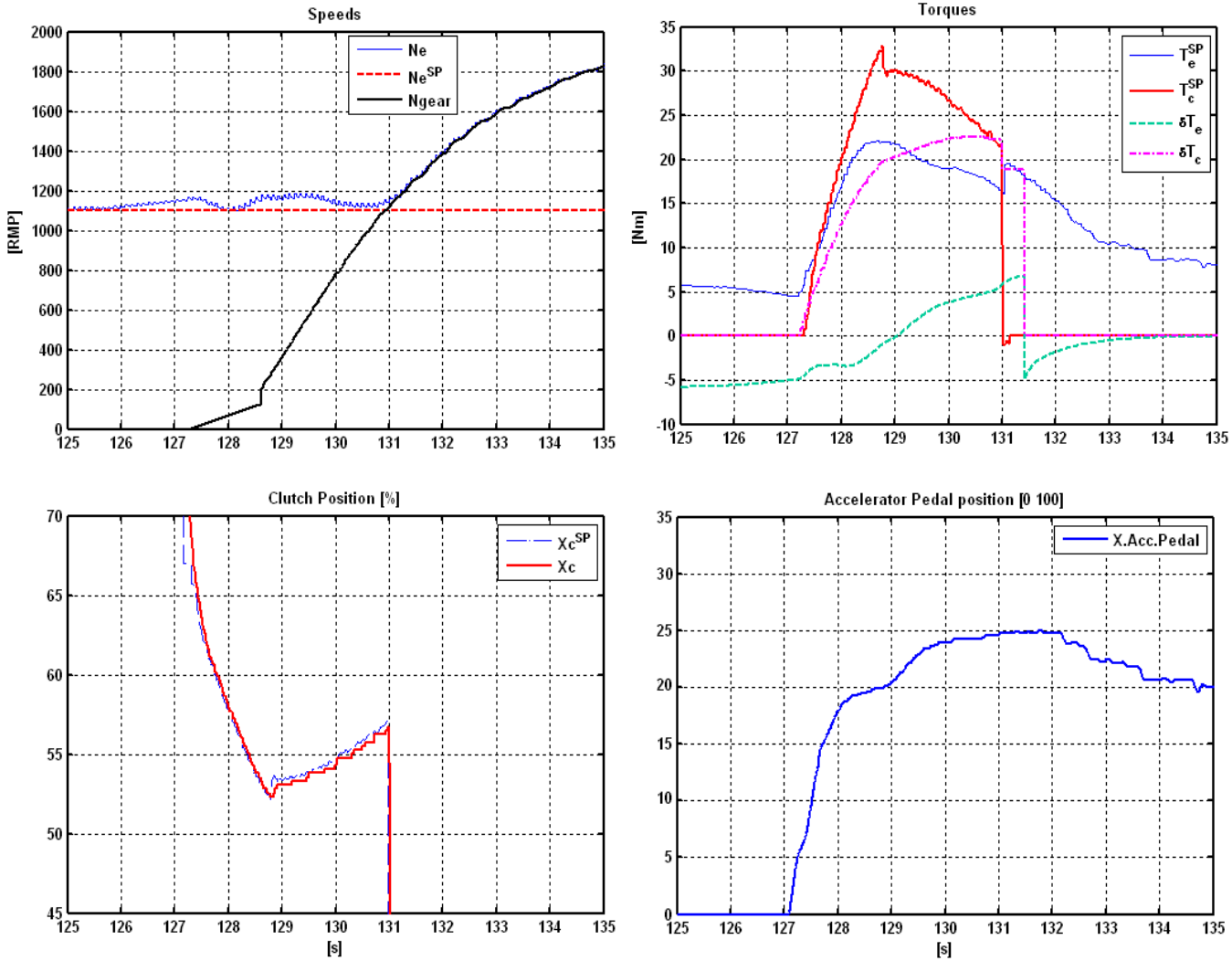
Comparison with pre-existing PID Controller (Fast start-up)



PID Controller

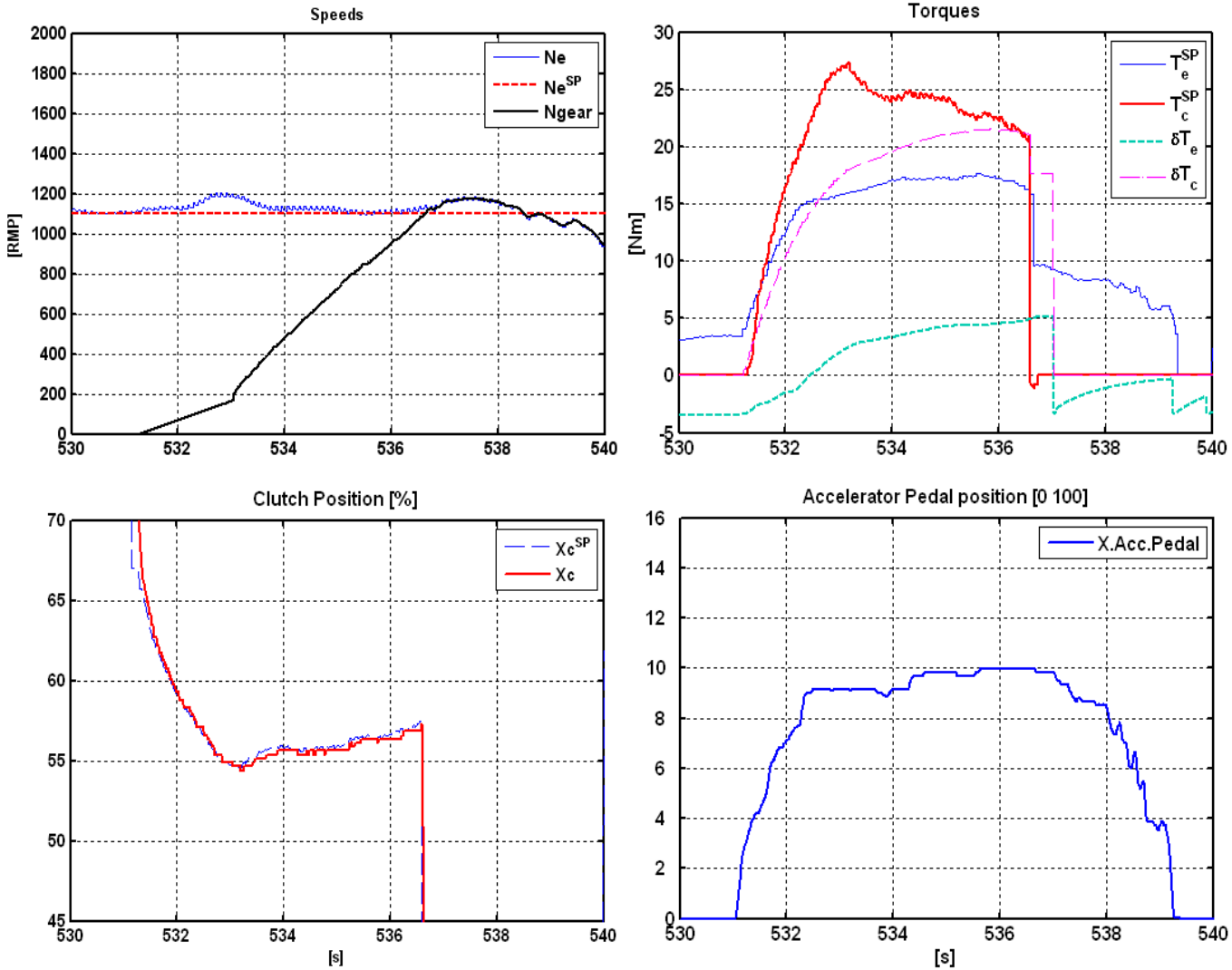
gipsa-lab

Comparison with pre-existing PID Controller (Fast start-up)



Parameterized NMPC

Comparison between slow and fast start-up



Parameterized NMPC - slow start-up

(The MPC scheme controls both the engine idle speed and the start-up phase)

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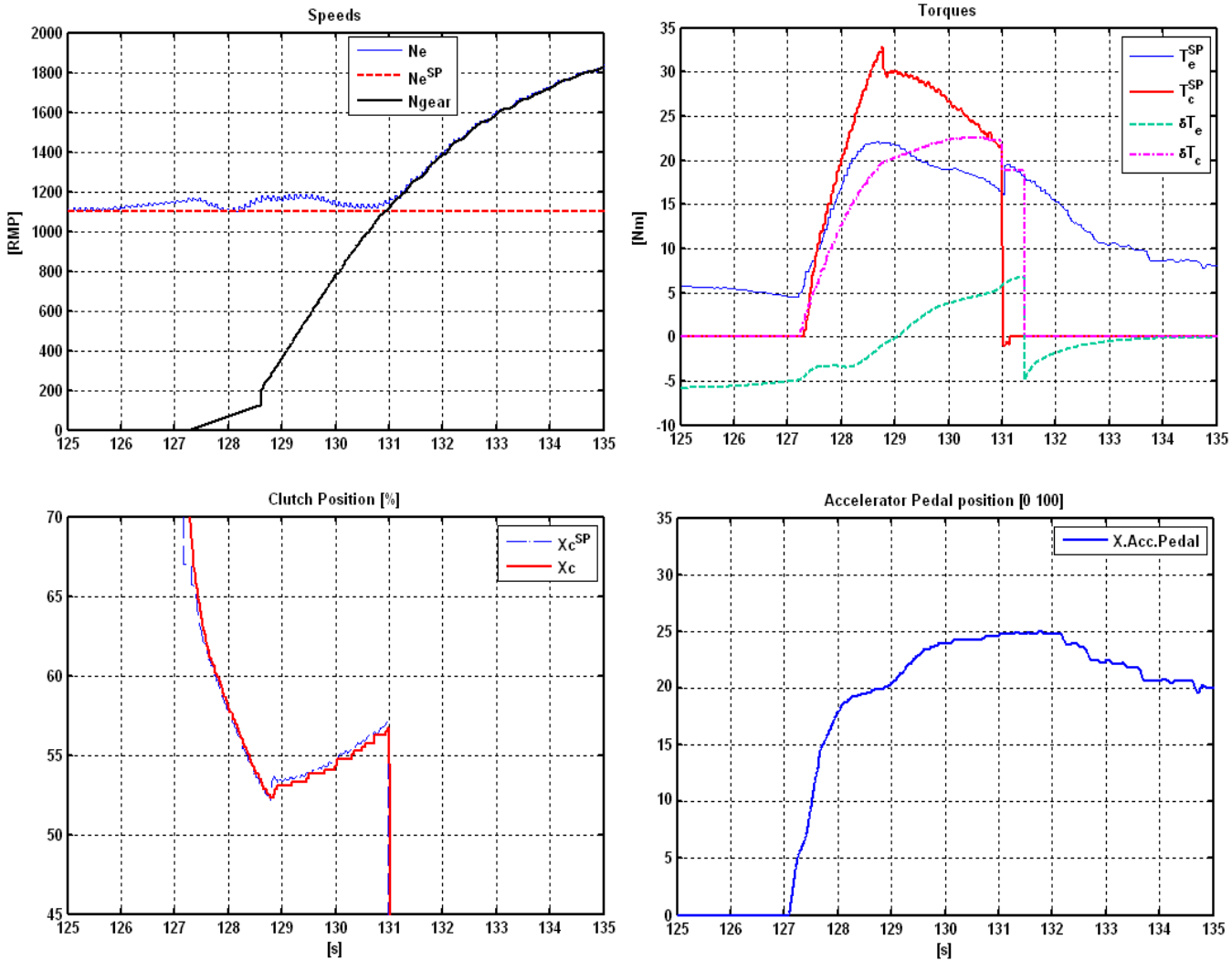
Comparison between slow and fast start-up



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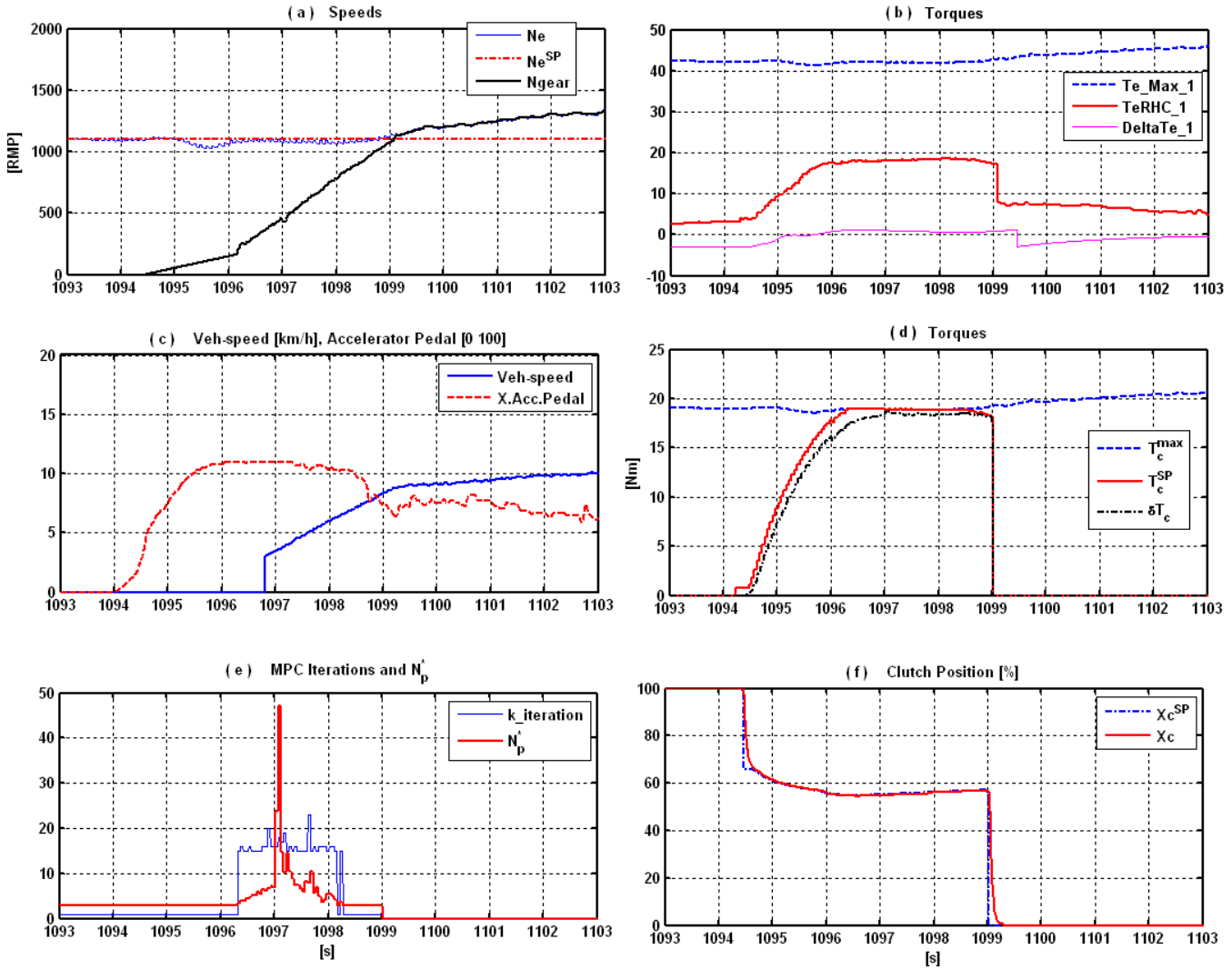
Behavior Under Saturation



Parameterized NMPC - Actuators saturation

(Note the closed-loop evolution of the parameter $p =: N_p^*$)

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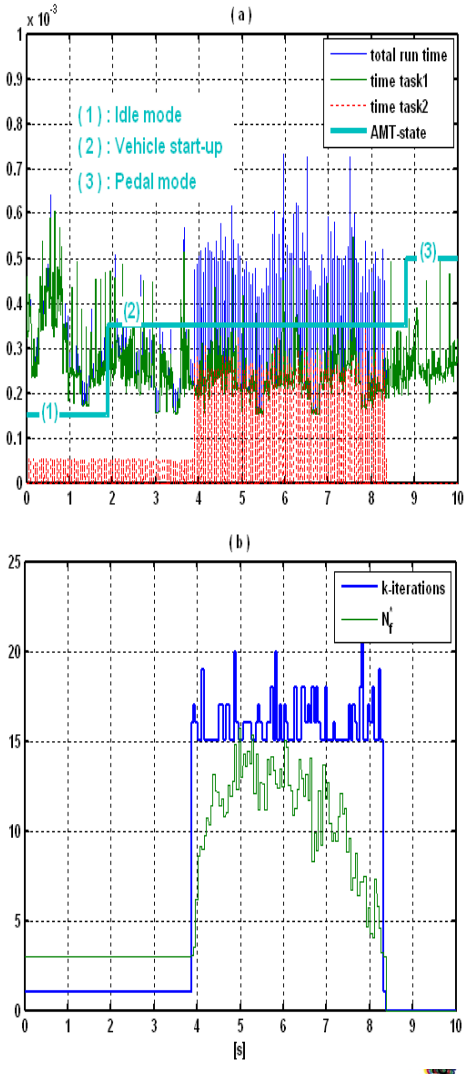
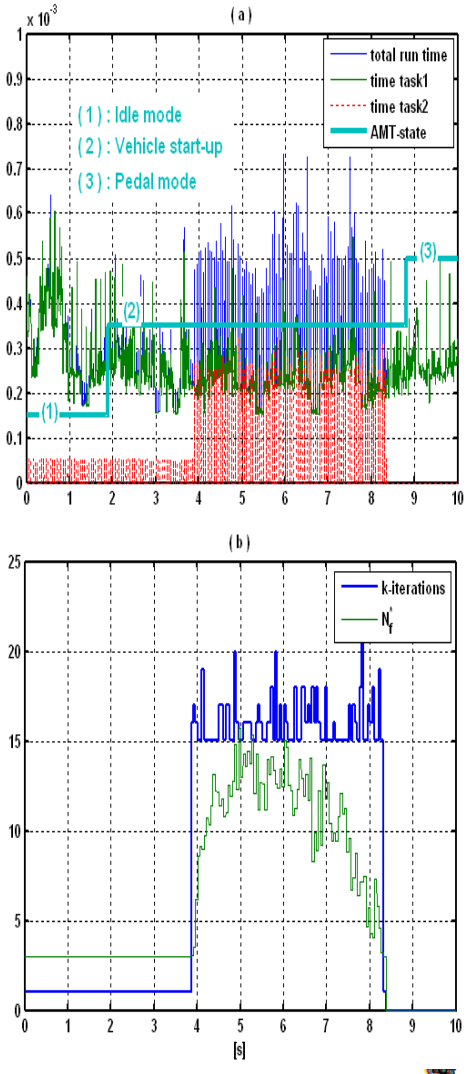
Some Real-Time Implementation Issues

- Matlab/Simulink, xPC Target



Some Real-Time Implementation Issues

- Matlab/Simulink, xPC Target
- Pentium III, 256 Mb (RAM)



Some Real-Time Implementation Issues

- Matlab/Simulink, xPC Target
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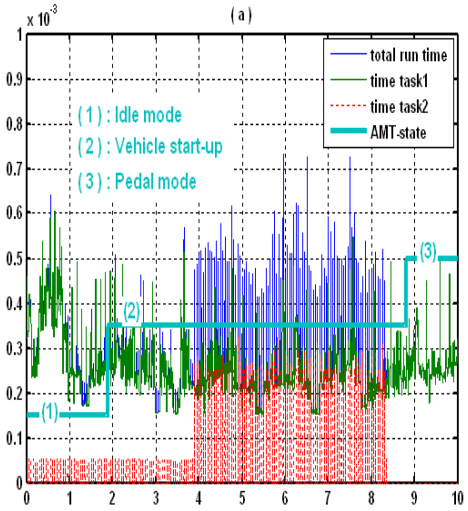
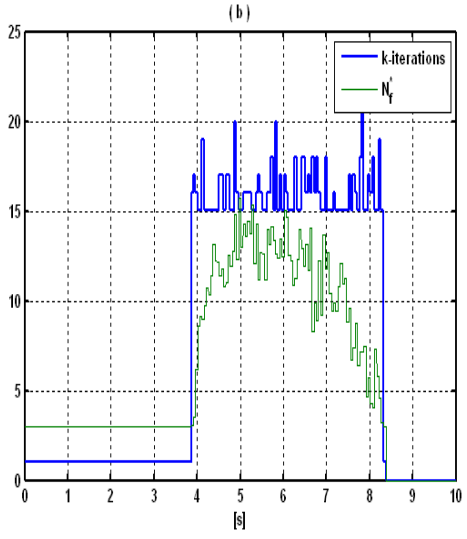
- Only 0.3Mb for Control Software





Some Real-Time Implementation Issues

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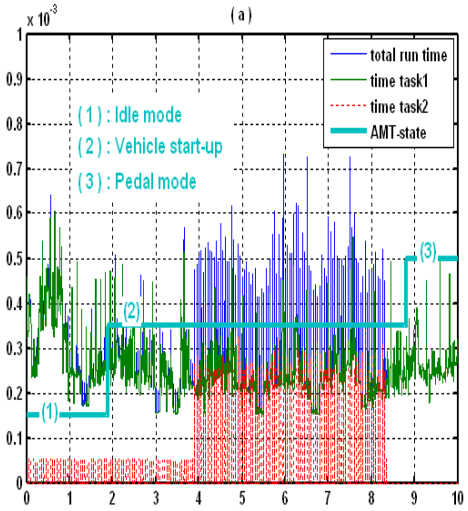
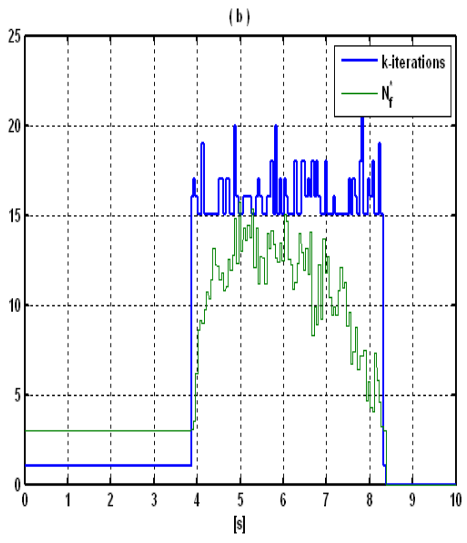
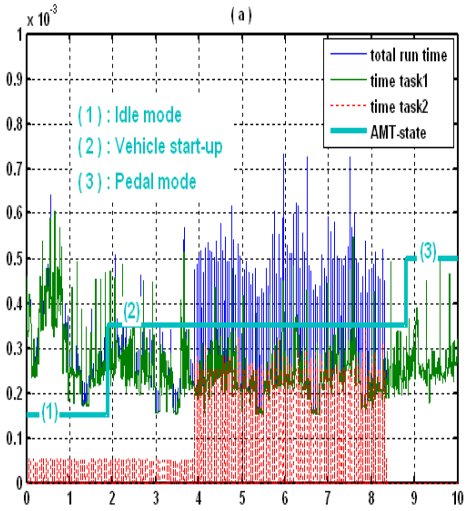
- Only 0.3Mb for Control Software
- Base sampling time 1 ms

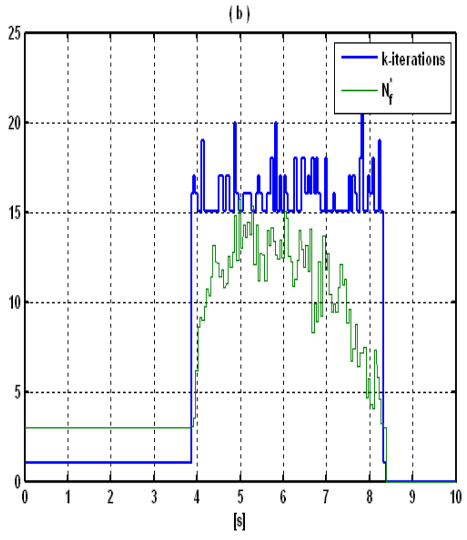




Some Real-Time Implementation Issues

- Matlab/Simulink, xPC Target
- Pentium III, 256 Mb (RAM)
- Only 0.3Mb for Control Software
- Base sampling time 1 ms
- Control blocks are executed synchronously with cylinder top dead center





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- Base sampling time 1 ms
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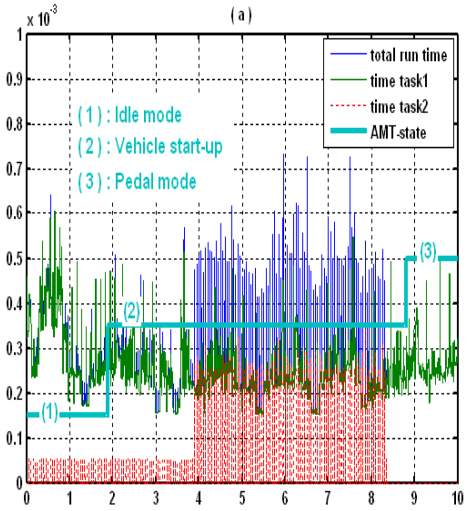

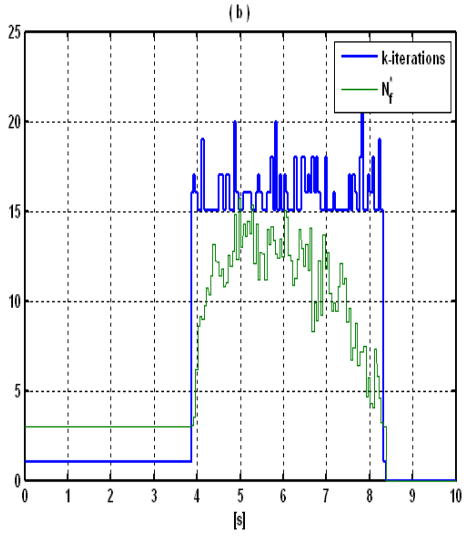

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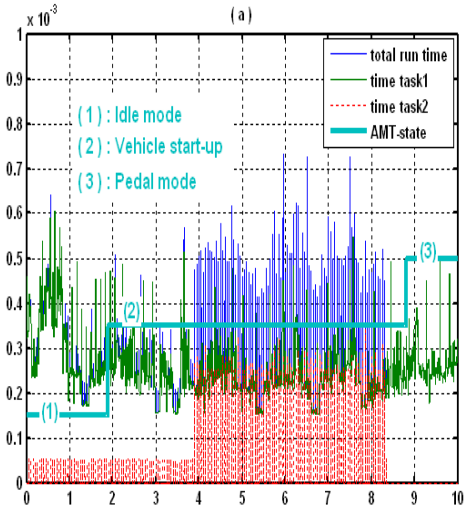

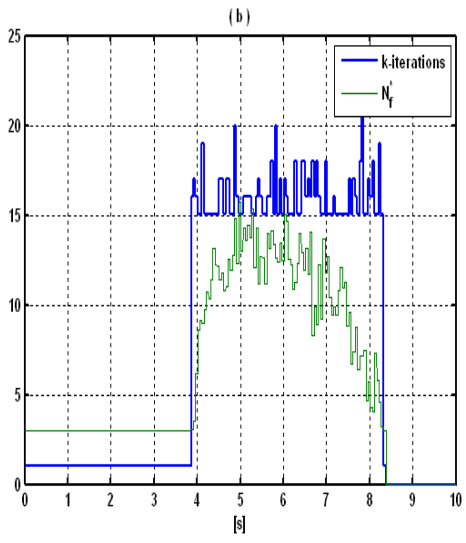

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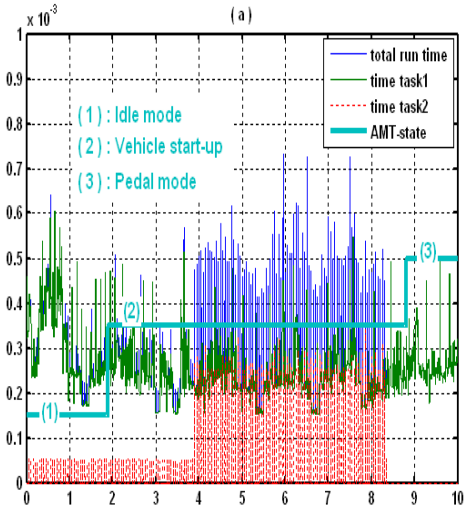

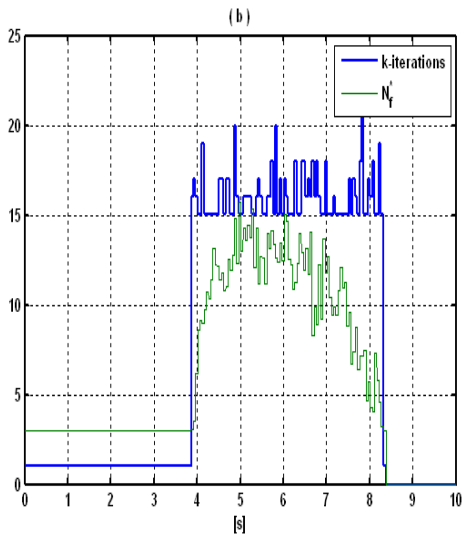

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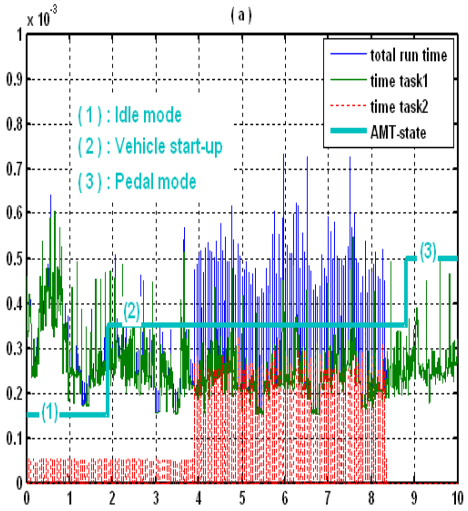

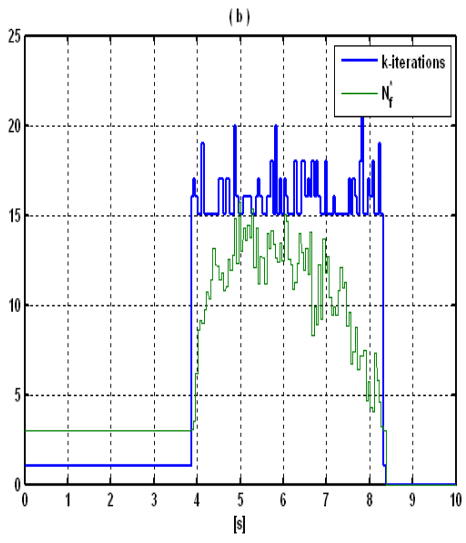

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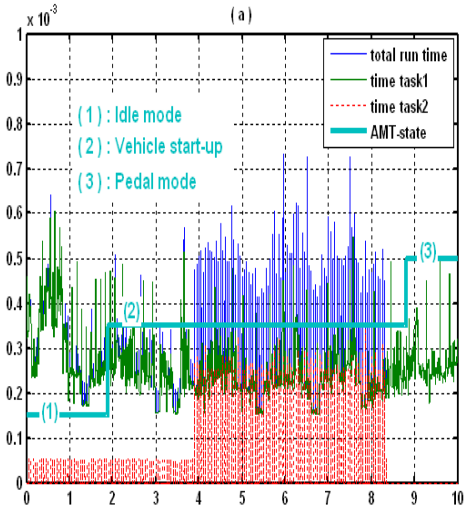

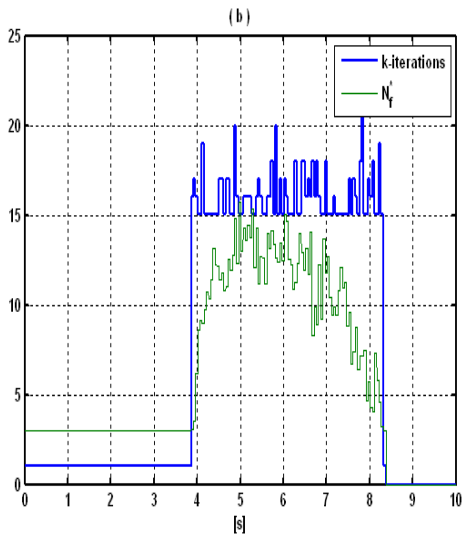

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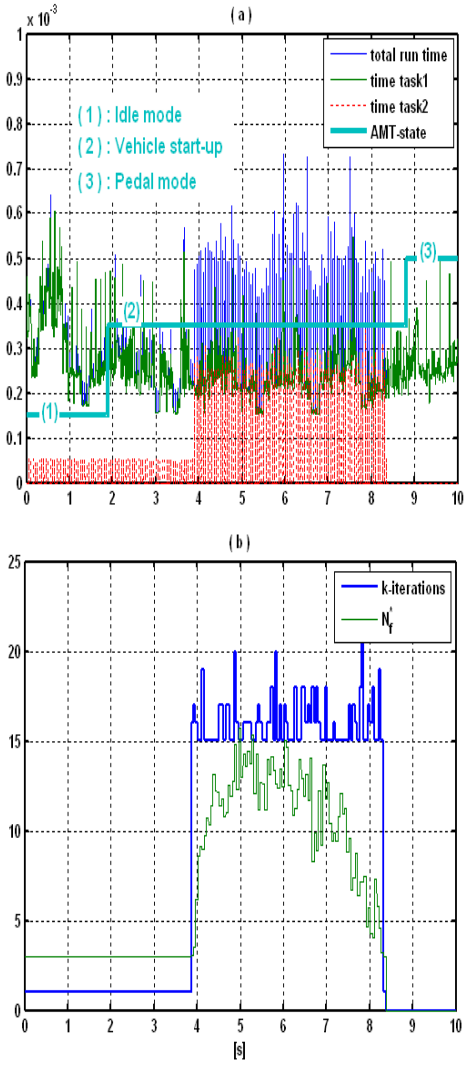

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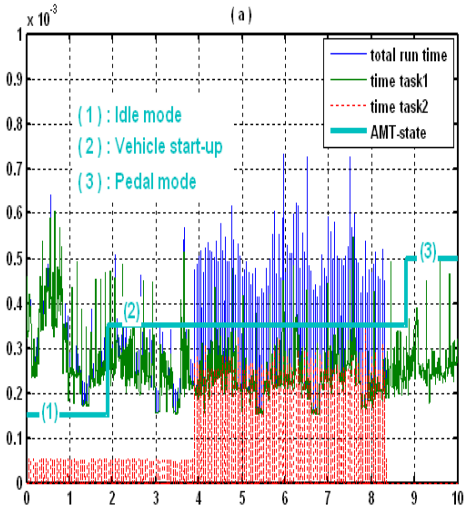

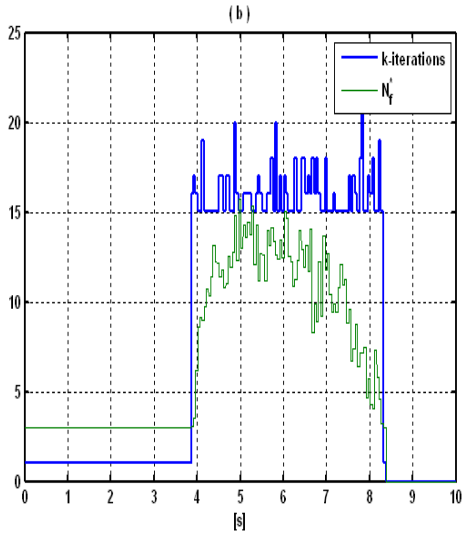

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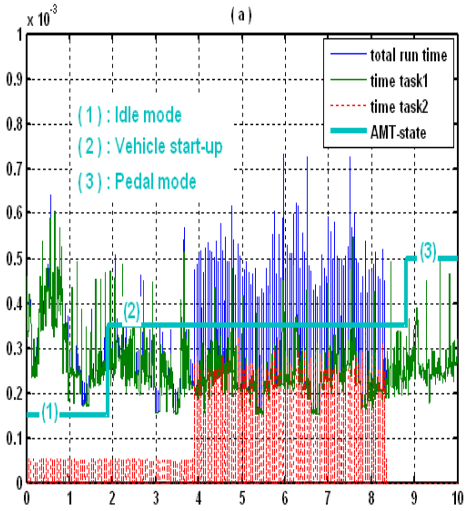

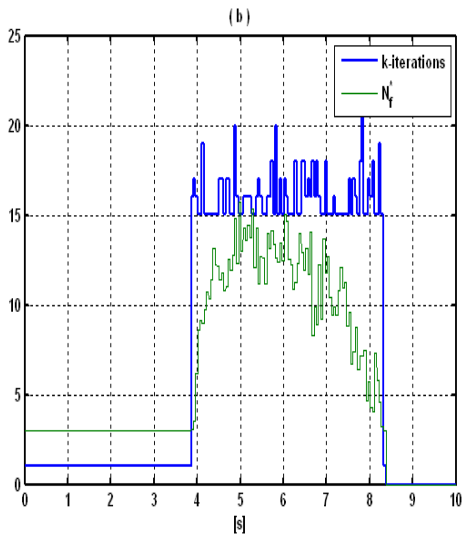

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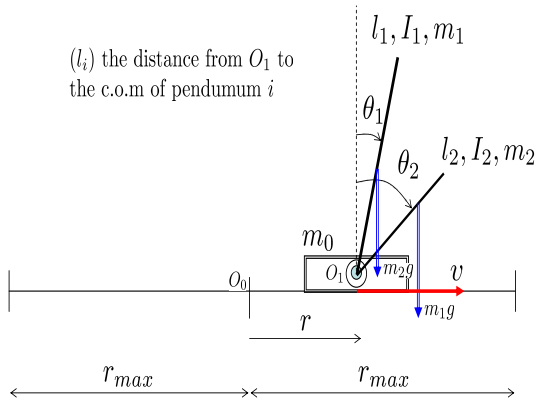

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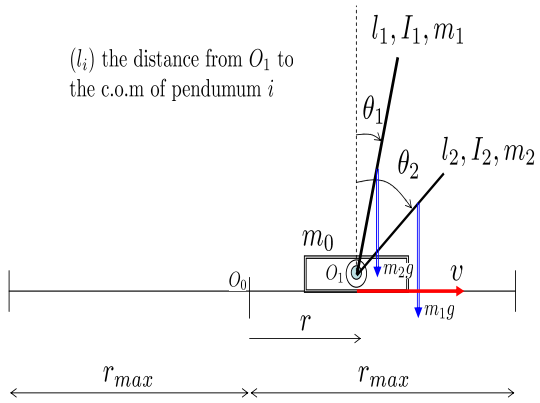

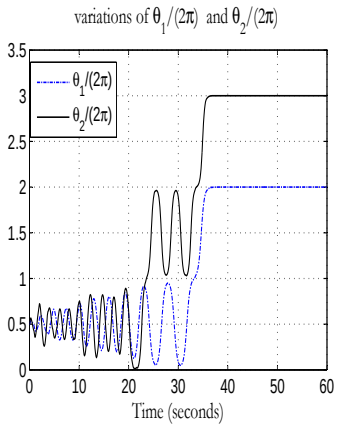

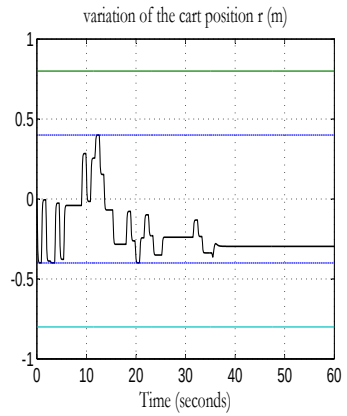

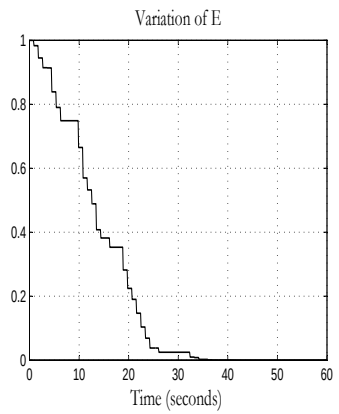

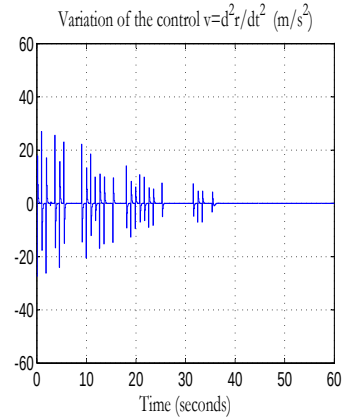

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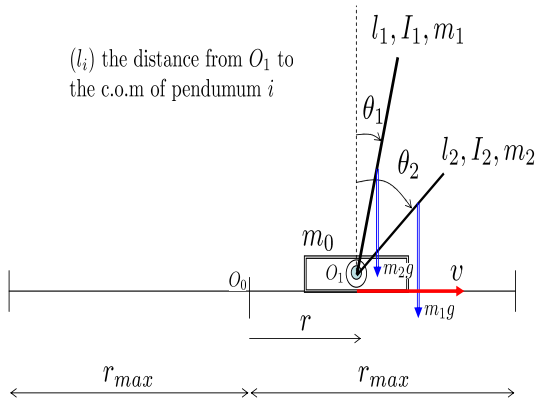

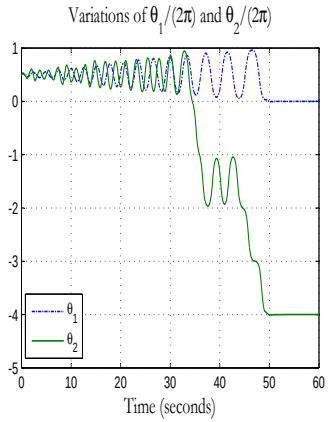

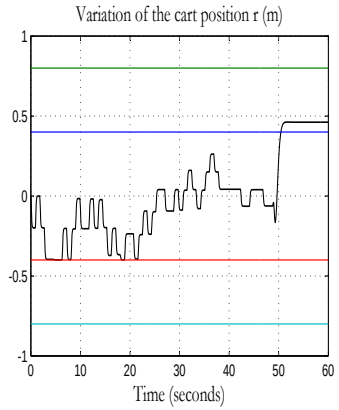

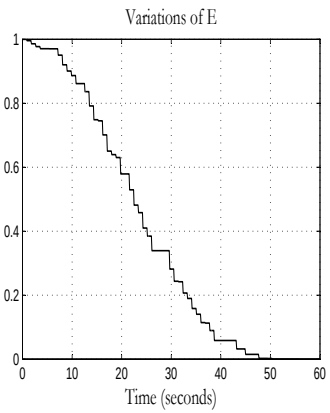

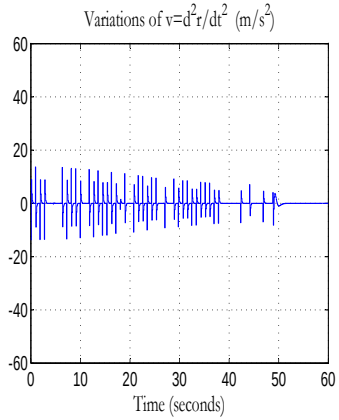

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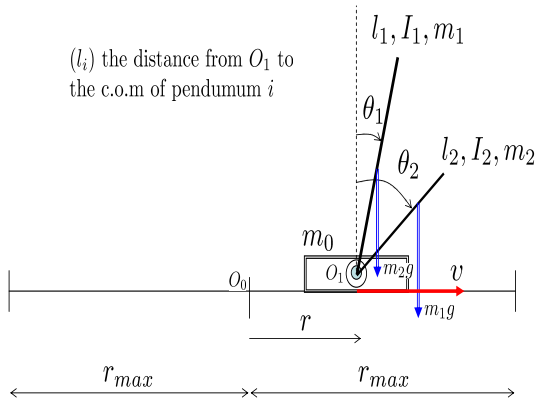

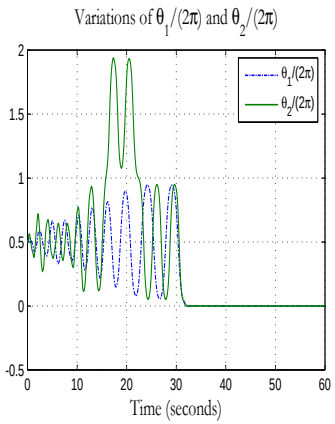

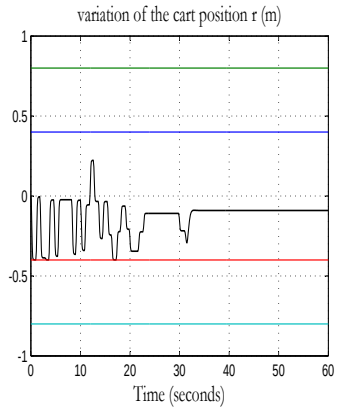

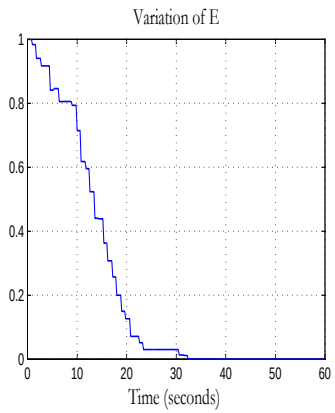

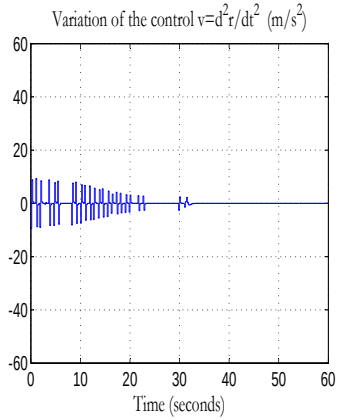

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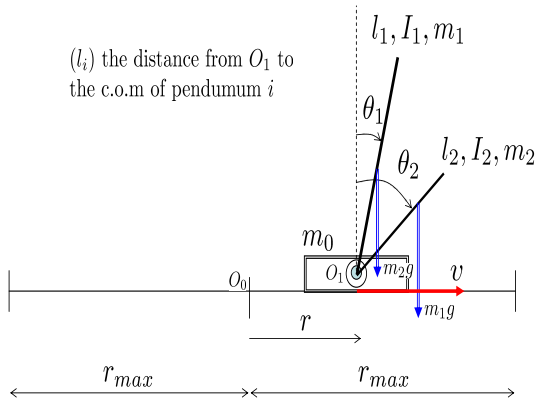

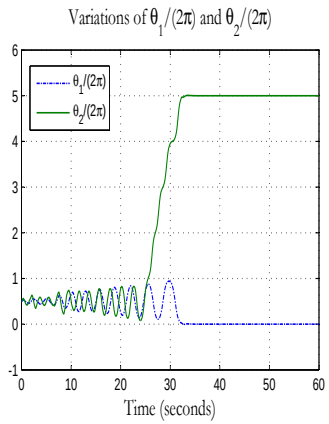

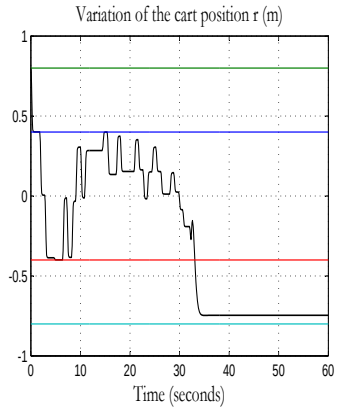

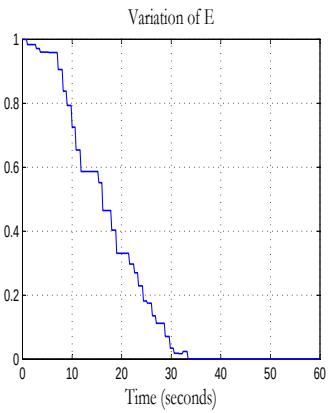

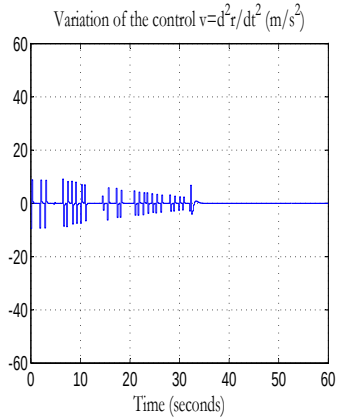

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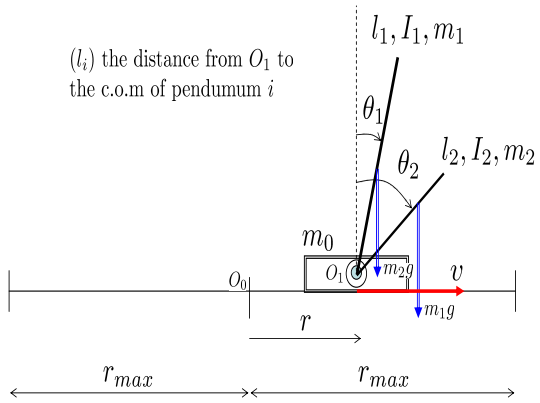

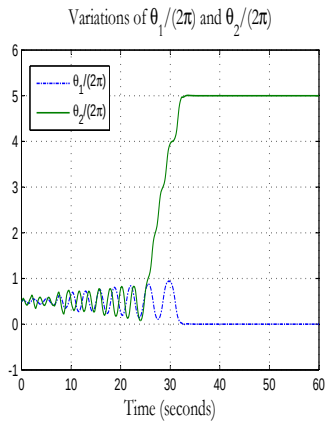

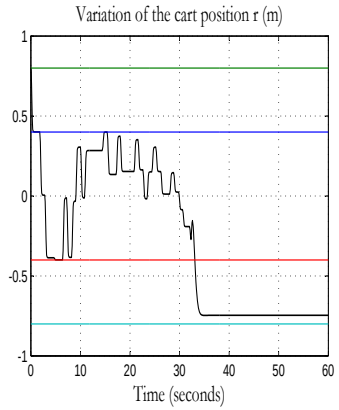

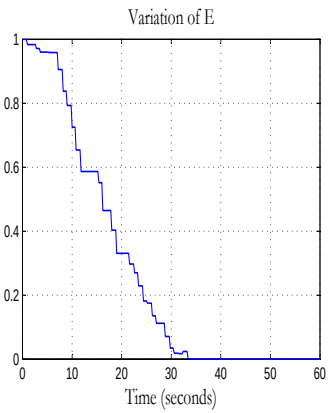

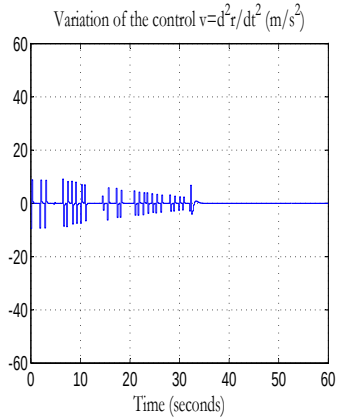

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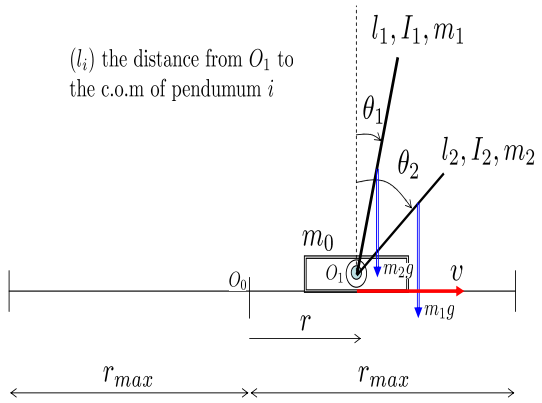

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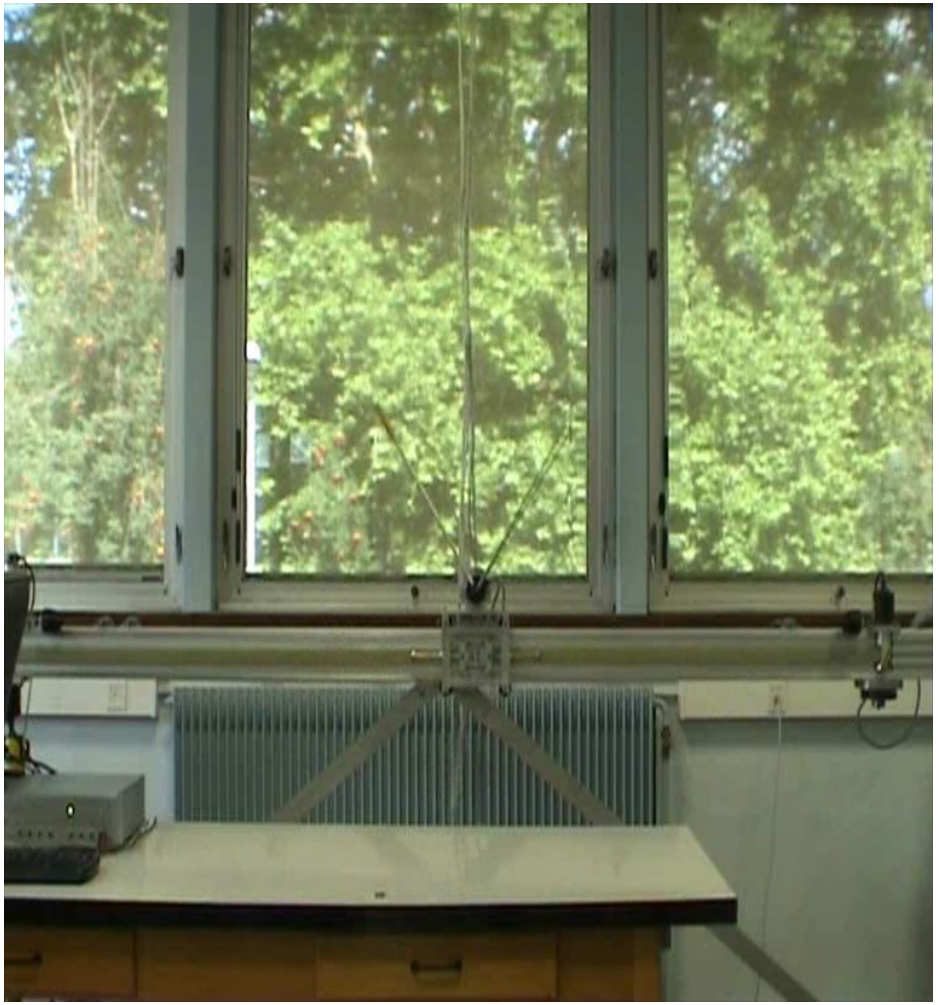

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- Space applications are definitely inside the feasibility domain of NMPC

Sufficient Conditions for Stability (but not necessary)

Necessary conditions for well defined and stable NMPC scheme

$$\mathcal{P}(x) \; : \; \min_{\mathbf{u}} \left\{ V(x,\mathbf{u}) \quad | \quad \mathbf{u} \in \mathcal{U}(x) \right\} \quad ; \quad x^+ = f(x,u)$$

$$\mathcal{U}(x) := \left\{ \mathbf{u} \in \mathbb{U}^N \mid \forall i \quad x^{\mathbf{u}}(i;x) \in \mathbb{X} \quad \text{and} \quad x^{\mathbf{u}}(N,x) \in X_f \right\}$$

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Condition 1 : Continuity

The applications f, F, L are continuous in their arguments.

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Condition 2 : Compactness

- ✓ $\mathbb{U}$ is compact
- ✓ $\mathbb{X}$ and $X_f \subset \mathbb{X}$ are closed.

Sufficient Conditions for Stability (but not necessary)

Necessary conditions for well defined and stable NMPC scheme

$$\mathcal{P}(x) \; : \; \min_{\mathbf{u}} \left\{ V(x,\mathbf{u}) \quad | \quad \mathbf{u} \in \mathcal{U}(x) \right\} \quad ; \quad x^+ = f(x,u)$$

$$\mathcal{U}(x) := \left\{ \mathbf{u} \in \mathbb{U}^N \mid \forall i \quad x^{\mathbf{u}}(i;x) \in \mathbb{X} \quad \text{and} \quad x^{\mathbf{u}}(N,x) \in X_f \right\}$$

$$V(x,\mathbf{u}) = F(x(N;x)) + \sum_{i=0}^{N} L(x^{\mathbf{u}}(i;x), u(i))$$

Condition 3 : Detectability

The integral cost L must be such that

$$\left\{ L(x,u) \to 0 \right\} \quad \Rightarrow \quad \left\{ x \to 0 \right\}$$

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$$V(x,\mathbf{u}) = F(x(N;x)) + \sum_{i=0}^{N} L(x^{\mathbf{u}}(i;x), u(i))$$

Condition 4 : X_f is positively invariant under some local $\kappa^f(\cdot)$

For all $\xi \in X_f$, there exists an admissible control $\kappa^f(\xi)$ such that :

$$f(\xi, \kappa^f(\xi)) \in X_f$$

psa-lab

Sufficient Conditions for Stability (but not necessary)

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$$V(x,\mathbf{u}) = F(x(N;x)) + \sum_{i=0}^{N} L(x^{\mathbf{u}}(i;x), u(i))$$

Condition 5 : The terminal cost $F(\cdot)$ is a Lyapunov under $\kappa^f(\cdot)$

For all $\xi \in X_f$, the admissible control $\kappa^f(\xi)$ is such that :

$$F(f(\xi, \kappa^f(\xi)) - F(\xi) \leq -L(\xi, \kappa^f(\xi))$$

psa-lab

Sufficient Conditions for Stability (but not necessary)

Necessary conditions for well defined and stable NMPC scheme

$$\mathcal{P}(x) \; : \; \min_{\mathbf{u}} \left\{ V(x,\mathbf{u}) \quad | \quad \mathbf{u} \in \mathcal{U}(x) \right\} \quad ; \quad x^+ = f(x,u)$$

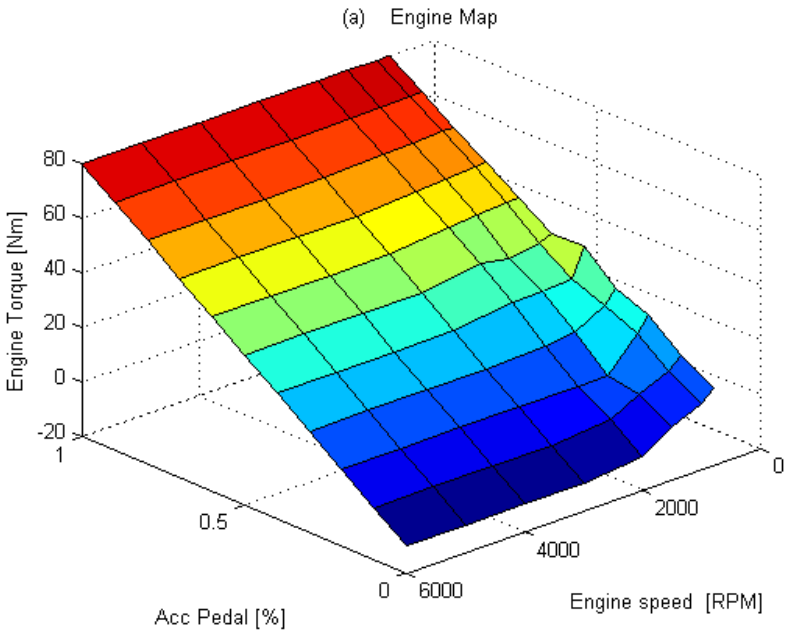
$$\mathcal{U}(x) := \left\{ \mathbf{u} \in \mathbb{U}^N \mid \forall i \quad x^{\mathbf{u}}(i;x) \in \mathbb{X} \quad \text{and} \quad x^{\mathbf{u}}(N,x) \in X_f \right\}$$

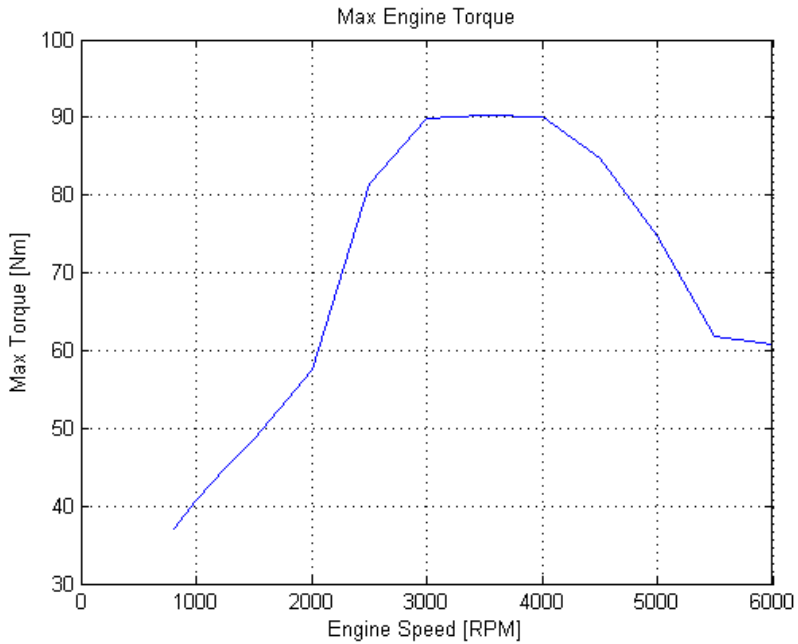
$$V(x,\mathbf{u}) = F(x(N;x)) + \sum_{i=0}^{N} L(x^{\mathbf{u}}(i;x), u(i))$$

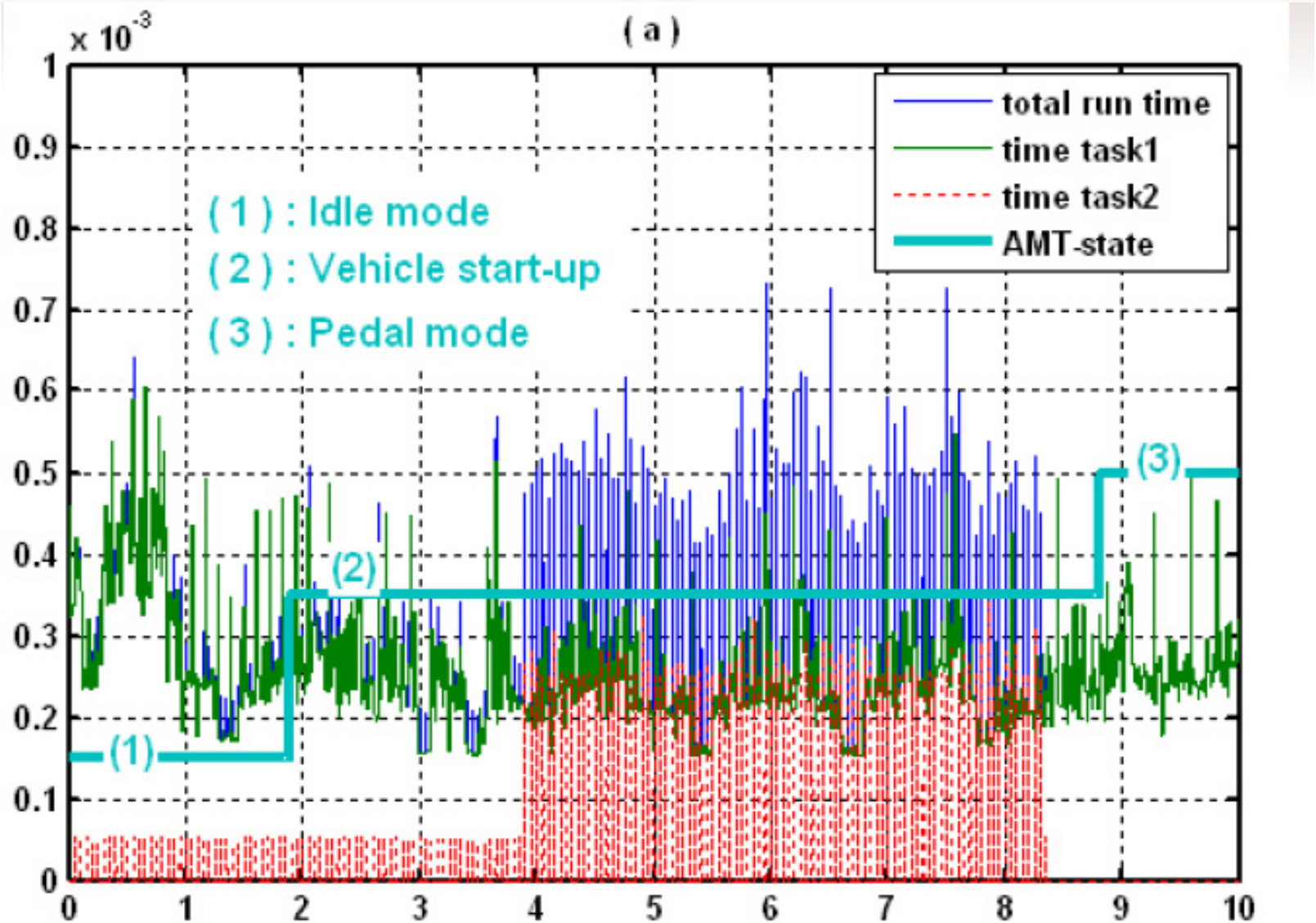
Condition 6 : Feasibility

There is at least a sequence that meets the constraints (in particular $x(0) \in X_N$ (the subset of state steerable in N steps to X_f with bounded controls in $\mathbb{U}$).

The IFP's SMART demo maps






(a)
x 10^-3
total run time
time task1
time task2
AMT-state
(1) : Idle mode
(2) : Vehicle start-up
(3) : Pedal mode
(1)
(2)
(3)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
1
2
3
4
5
6
7
8
9
10