Output Feedback Control of a Class of Uncertain Systems Under Control-derivative Dependent Disturbances

M. Alamir, J. Dobrowolski and A. T. Mohammed

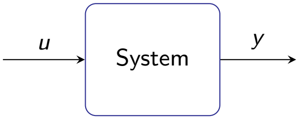
CNRS / Gipsa-lab/ University of Grenoble-Alpes

Schneider-Electric Industrie

General-Electric

Problem Statement

- Control an output y using input u (y is of relative degree 1)



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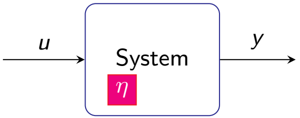

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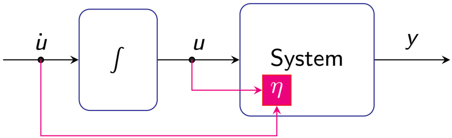

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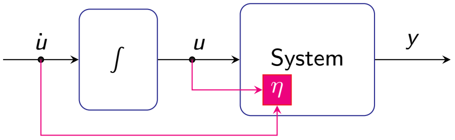

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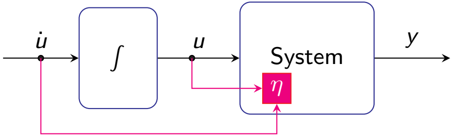
Lack of observability

Badly known dynamics

Industrial structure



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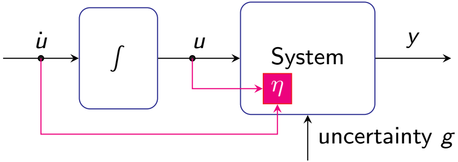

Problem Statement

Tune the parameters (λ, λ_f) of a simple dynamic output feedback law:

$$\dot{z} = \lambda_f(u - z)$$
$$u = Sat\Big(\lambda(y_d - y) + z\Big)$$

such that stability is guaranteed despited of η and g.

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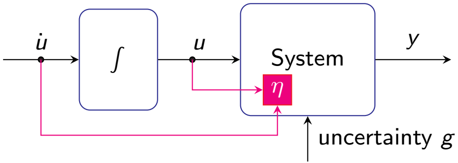
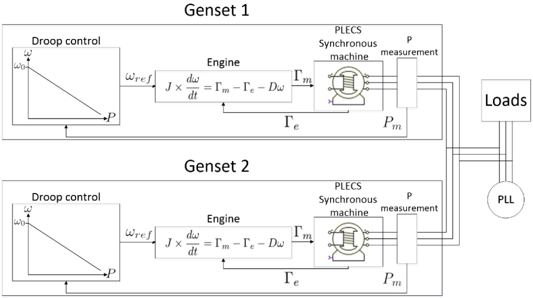


Motivating Examples (1): Frequency control in microgrids

$y \leftrightarrow f$		(grid frequency)
$u \leftarrow P_p$		produced power
$\eta \leftarrow$		coupling harmonics
$w \rightarrow$		load power



[See Dobrowolski et al. at this conference]

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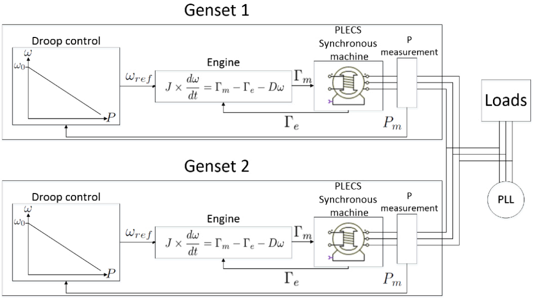

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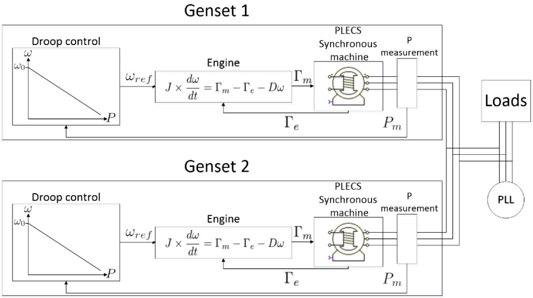

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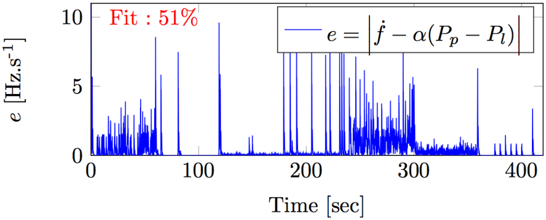
Checking the equation $\dot{f} = \alpha(P_p - P_l)$



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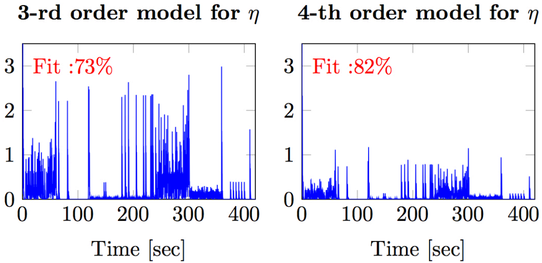

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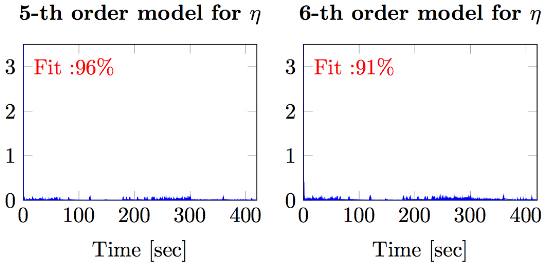

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Motivating Examples (2): Hydraulic Power Stations

$y \leftrightarrow \Omega$	(rotation speed)
$u \leftarrow Q$	flow rate at turbine's level
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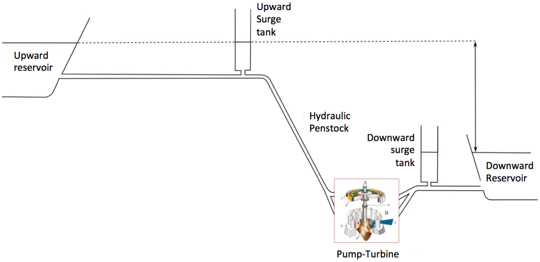
$w \rightarrow$	unknown dynamics



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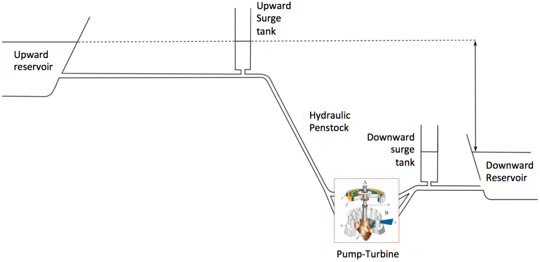
$$\dot{\Omega} = \alpha(T(H, Q, \Omega) - w)$$

$$\texttt{Hyperbolic}(H(\cdot), Q(\cdot)) = 0$$



Working Assumptions

$$\dot{y}(t) = \alpha(t)\Big[u(t) - g(t) + \ell(t, \eta(t))\Big]$$
$$\dot{\eta}(t) = E(\boldsymbol{\eta}^{[t-\tau,t]}, \dot{\mathbf{u}}^{[t-\tau,t]}, \mathbf{w}^{[t-\tau,t]})$$

- ✗ g unmeasured disturbances
- ✗ η unmeasured state
- ✗ $\ell(\cdot), E(\cdot)$ unknown maps

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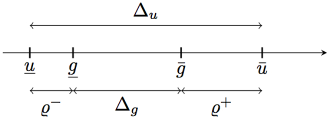
- ✓ $g \in [\underline{g}, \bar{g}]$, $|\dot{g}| \le \delta_g$
- ✓ $u \in [u, \bar{u}]$ is such that:



- ✓ $|\ell(t, \eta(t))| \le c_0 + c_1 \|\dot{\mathbf{u}}^{[t-\tau,t]}\|_\infty$
- ✓ $|\dot{\ell}(t, \eta(t))| \le d_0 + d_1 \|\dot{\mathbf{u}}^{[t-\tau,t]}\|_\infty$
- ✓ Dynamics of η is open-loop stable.

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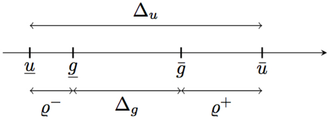

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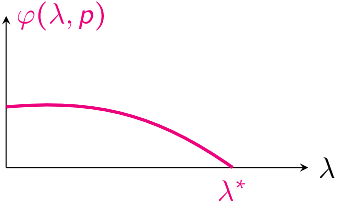
Result expressed using $p := \{\underline{\alpha}, \bar{\alpha}, \varrho^-, \varrho^+, \Delta_u, \delta_g, c_0, d_0, c_1, d_1\}$.

Main result

If $c_0 < \min\{\varrho^-, \varrho^+\}$ then $\exists\ \varphi(\lambda, p)$ (explicitly defined):



$$\dot{z} = \lambda_f (u - z) \quad (1)$$

$$u = Sat\Big(\lambda(y_d - y) + z\Big) \quad (2)$$

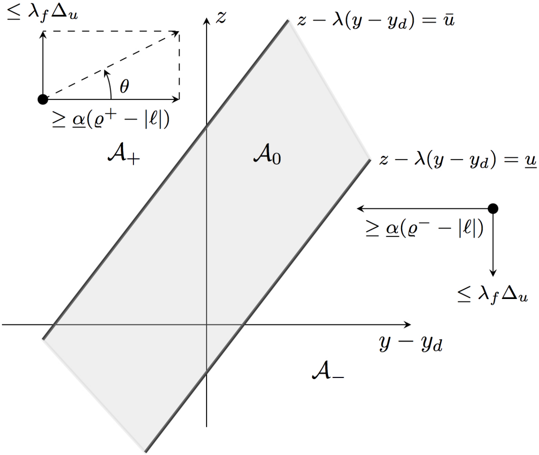
$\forall(\lambda, \lambda_f)$ such that:

1. $\lambda \in (0, \lambda^*)$
2. $\lambda_f \leq \lambda \cdot \varphi(\lambda, p)$

the CL system under (1)-(2) is stable and satisfies:

$$\lim_{t\to\infty} |y(t) - y_d| \leq \frac{\varepsilon(p)}{\lambda\lambda_f}$$

Main Idea Behind the Proof



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$\dot{\eta}(t) = A\eta(t) + B_1\|\dot{\mathbf{u}}^{[t-\tau,t]}\|_\infty + B_2 w$
$\ell(\eta) = C\eta$
$g(t) = 1 - 0.3\cos(0.1t) - 0.1\sin(0.2t + \pi/6)$
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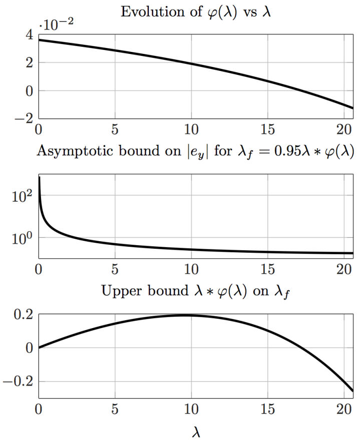

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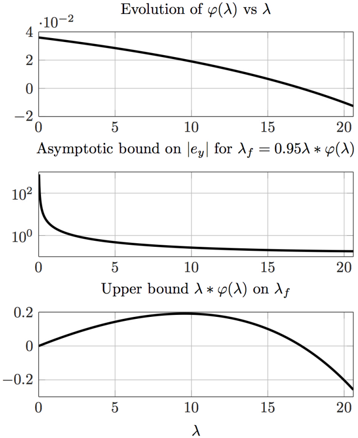
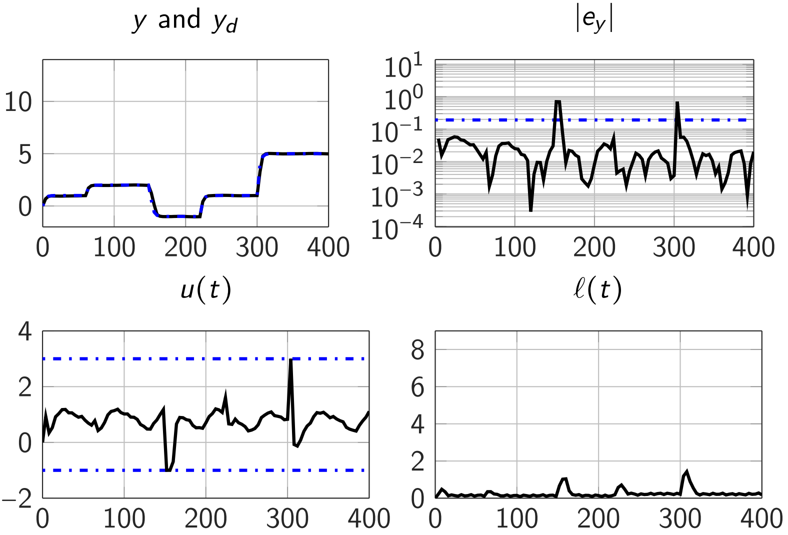
$\rightarrow \lambda^* = 17.2$

$\rightarrow \lambda = 17$

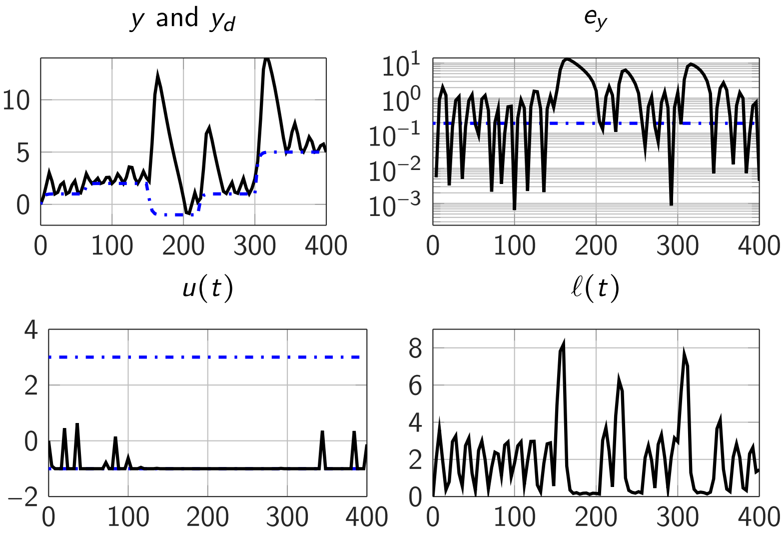
$\rightarrow \lambda_f = 0.95\lambda\varphi(\lambda, p) = 0.01$



Simulation with the compute values $(\lambda, \lambda_f) := (17, 0.01)$



Simulation with the compute values $(\lambda, \lambda_f) := (170, 0.01)$



Application on Microgrids is detailed in:

16:40-17:00, Paper TuP19.3

J. Dobrowolski, M. Alamir, S. Bacha, D. Gualino and M. X. Wang. On higher order dynamics in the fundamental equation of frequency in islanded microgrids. Proceedings of the IFAC World Congress, Toulouse, France, 2017.

Recent experimental results show high oscillations as soon as $\lambda > 1.6\lambda^*$.