Nonlinear Model Predictive Control

Slow Thoughts for Fast Implementation

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Outline

- Control Parametrization in NMPC
- Pros & Cons
- Real-Time Implementation
- Example 1 : Diesel Engine Control
- Example 2 : Automated Manual Transmission
- Conclusion

Control Parametrization

Consider a dynamical system :

$$x(t) = X(t, x_0, \mathbf{u}) \quad x \in \mathbb{R}^n \quad \mathbf{u} \in \mathbb{U}^{[0,T]}$$

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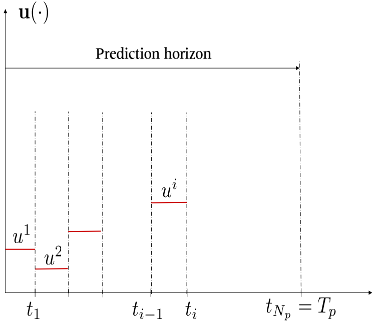
$$\mathbf{u}(t) = u^{(k)}(p) \quad ; \quad t \in [t_{k-1}, t_k]$$



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More generally

Any map

$$C : \mathbb{P} \rightarrow \mathbb{U}^N$$
$$C(p) = \begin{pmatrix} u^1(p) & \dots & u^N(p) \end{pmatrix}$$

defines a $\mathbb{P}$-parameterized piecewise constant control profile

$$\mathbf{u} = \mathcal{U}_{pwc}(\cdot, p)$$

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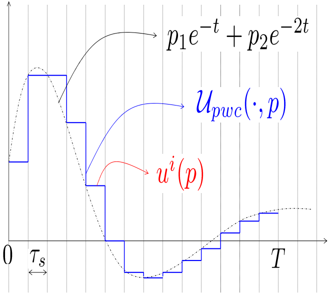

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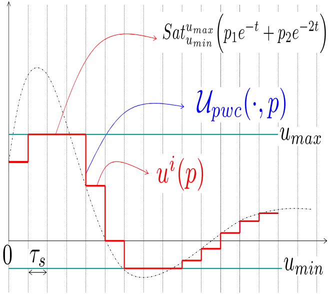

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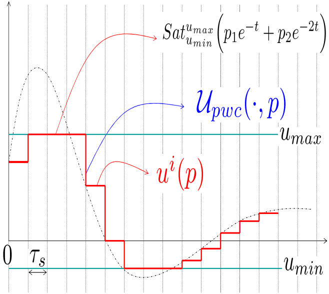


NMPC

$$K := u^{(1)} \circ \hat{p} : \mathbb{R}^n \rightarrow \mathbb{U}$$

$$\hat{p}(x(t_j)) := \arg\min_{p \in \mathbb{P}} \Big[J(x(t_j), p) \Big]$$

$$\text{under} \quad C(x(t_j), p) \leq 0$$

Control Parametrization : Pros & Cons

Fact 1

Simple OL param.

$\downarrow$

Complex CL behavior

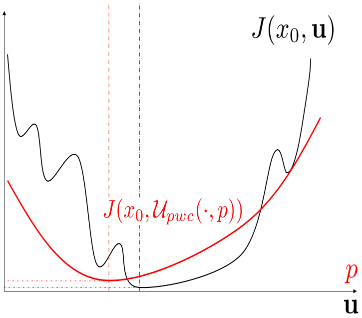
Control Parametrization : Pros & Cons

Fact 2

Solve exactly a sub-optimal formulation

could be better than

Solve loosely the exact problem



Control Parametrization : Pros & Cons

Fact 3

Reduced dimensional and well posed optimization problems

Render efficient

A family of simple, non smooth and memoryless algorithms that would be out of scope for large scale problems

Control Parametrization : Pros & Cons

Fact 4

Low dimensional formulation & simple self-contained algorithms are more likely to be assessed and hence admitted in industrial context.

Control Parametrization : Pros & Cons

Fact 5

- Problem dependent

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[Alamir, M. Stabilization of Nonlinear Systems Using Receding-Horizon Control Schemes : A Parametrized Approach for Fast Systems. Lecture Notes in Control and Information Sciences, Springer, London, ISBN 1-84628-470-8 (2006)]

Real-Time Implementation

NMPC

$$K := u^{(1)} \circ \hat{p} : \mathbb{R}^n \to \mathbb{U}$$

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Iterative process $\mathcal{S}$

$$p^{(i+1)} = \mathcal{S}(p^{(i)}, x(t_k))$$

$$p^{(0)} = p_0 \in \mathbb{P}$$

- Choose a sampling period τ_s
- Deduce a maximum number of iterations $q \in \mathbb{N}$
- Distribute the optimization over the system real life-time :

$$x(t_{k+1}) = X(\tau_s, x(t_k), u^1(p(t_k)))$$

$$p(t_{k+1}) = \mathcal{S}^{(q)}(p^+(t_k), \hat{x}(t_{k+1}))$$

Diesel Engine Control

Control of emission needs to track the following outputs

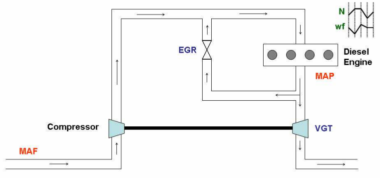
- Manifold Air Pressure (MAP)
- Mass Air Flow (MAF)

Control inputs

- Exhaust Gaz Recirculation (EGR)
- Variable Geometry Turbocharger (VGT)

Measured disturbances

- Engine Speed (N)
- Fuel Injection (wf)



Models & State Observer

Model 1 [tested by simulation]

$$
\begin{aligned}
x_1^+ &= [A(y)]x_1 + B_1 u + G_1 w \\
y_1 &= [C(y)]x_1
\end{aligned}
$$

- $x_1 \in \mathbb{R}^{n=13}$
- Sampling Time $\tau_s = 10\ ms$

Model 2 [tested experimentally (see later)]

$$
\begin{aligned}
x_2^+ &= [A(u,w)]x_2 + B_2 u + G_2 w \\
y_2 &= [C(u,w)]x_2
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- $x_2 \in \mathbb{R}^{n=8}$
- Sampling Time $\tau_s = 50\ ms$

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Moving Horizon Observer :

$$\hat{x}_k = \big[\Phi(\bar{U}_k)\big]\cdot \xi_{opt}(\bar{U}_k, \bar{Y}_k, \hat{x}_{k-1}) + \big[\Psi(\bar{U}_k)\big]\cdot \bar{U}_k$$

where :

$$\xi_{opt}(\bar{U}_k, \bar{Y}_k, \hat{x}_{k-1}) \quad \text{solution of a QP}$$

and where State/Output noise classical trade-off is introduced.

Diesel Engine : Control Parametrization

- y_d : desired output
- w : measured disturbance

$$\mathbf{u}(t) = Sat_{u_{min}}^{u_{max}}\left(u^*(y_d, w, u_d) + \alpha_1 e^{-\lambda t} + \alpha_2 e^{-q_c \lambda t}\right) \quad ; \quad \alpha_i \in \mathbb{R}^2$$

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- λ and q_c constant coefficients (Control bandwidth)

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- Stationary Control $u^*(y_d, w, u_d) = arg \min_{u \in [u_{min}, u_{max}]} \|y_c(u, w) - y_d\|^2$

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1. Continuity of the control ($t = 0$)

   $u^* + \alpha_1 + \alpha_2 = u_{k-1}$

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1. Continuity of the control ($t = 0$)

   $u^* + \alpha_1 + \alpha_2 = u_{k-1}$

2. Parametrization of the derivative ($t = \tau$)

   $(e^{-\lambda\tau} - 1) \cdot \alpha_1 + (e^{-q_c\lambda\tau} - 1) \cdot \alpha_2 = p\delta_{max}$ where $p \in [-1, 1]^2$

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$$\begin{pmatrix} \alpha_1(p) \\ \alpha_2(p) \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ e^{-\lambda\tau} - 1 & e^{-q_c\lambda\tau} - 1 \end{pmatrix}^{-1} \begin{pmatrix} u_{k-1} - u^*(y_d, w, u_d) \\ p\delta_{max} \end{pmatrix}$$

$$p \in \mathbb{P} = [-1, +1]^2$$

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- 2-dimensional problem that structurally meets the constraints.

Diesel Engine : The Cost Function & Control Parameters

$$J(p) := \rho_x \cdot \|x(N_p) - x^*\|^2 + \sum_{i=1}^{N_p} \Big[\|y(i) - y_d\|^2 \Big]$$

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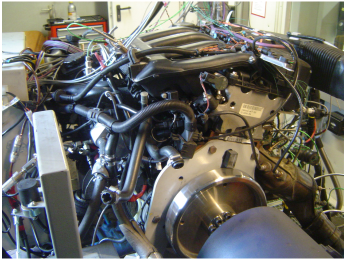
- Testbench Linz (BMW M47D Diesel Engine)
- Sampling time $\tau = 50$ ms
- Prediction Horizon $30 \cdot \tau$
- Observation Horizon $N_O = 10\tau$
- Solver SQP/Trust region
- Number of iteration $q = 30$
- Control param. $\lambda = 1$, $q_c = 5$



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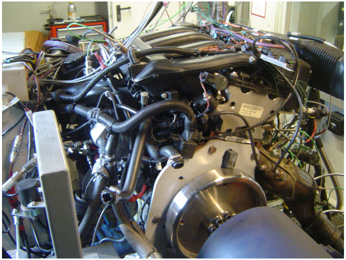
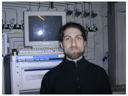


Preliminary experiments

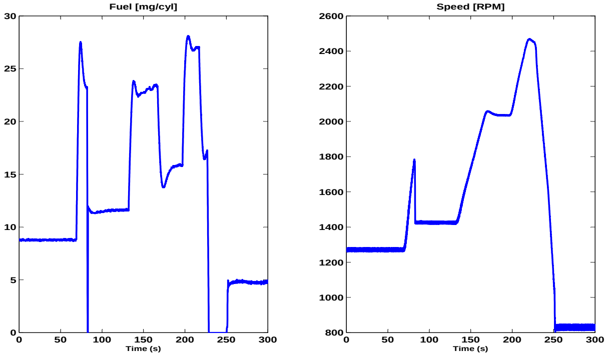

André Murilo



R. Fŭrhapter


Peter Ortner

Experimental Results



NMPC, Speed N and fuel injection w_f of the (New European Driving Cycle)

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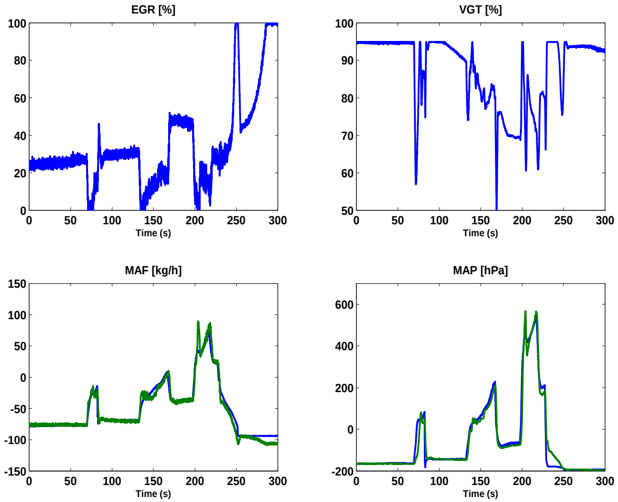

NMPC, norm. cycle

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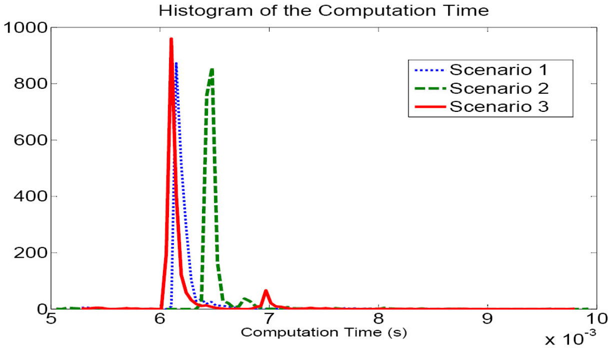
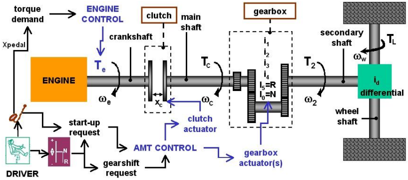


Fig. 7. Histograms of the computation time needed at at each sampling period to perform the observation task and the N_{iter} iterations of the optimization procedure.

Automated Manual Transmission

Trade-off between

- Manual : efficient, low weight, low cost
- Automatic : better comfort / high cost & consumption



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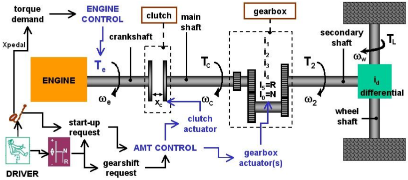
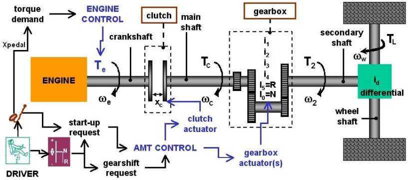


Control objective :

Automated Manual Transmission



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Control objective :

- Smooth transitions for $\omega_{sl} \rightarrow 0$

  (this is intimately linked to the dynamic of the slip velocity $\omega_{sl} = \omega_e - \omega_c$)

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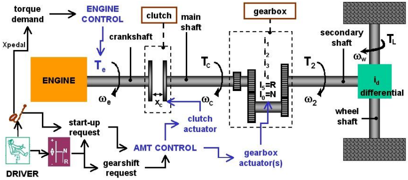

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  The torque must be somehow related to the pedal's position

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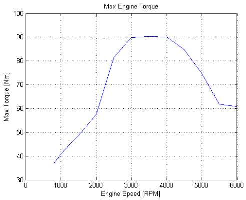

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  The torque must be somehow related to the pedal's position
- Control of the engine velocity :

$$\omega_e^{ref} = max\left\{\omega_e^0, \mathcal{T}^{-1}\left(T_e^d(X_{pedal}, \omega_e)\right)\right\}$$

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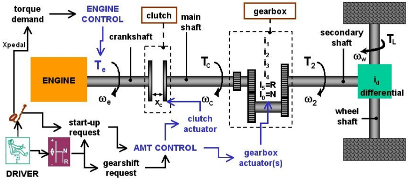

Constraints :

- Torque saturation

$$
\begin{aligned}
T_e &\in [T_e^{min}, T_e^{max}(\omega_e)] \\
T_c &\in [T_c^{min}, T_c^{max}(\omega_e)]
\end{aligned}
$$

- Torque variation rate saturation

$$
\begin{aligned}
\dot{T}_e &\in [\dot{T}_e^{min}, \dot{T}_e^{max}] \\
\dot{T}_c &\in [\dot{T}_c^{min}, \dot{T}_c^{max}]
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Simplified Model for Control

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$$J_e\dot{\omega}_e = T_e - sign(\omega_{sl}) \cdot T_c(x_c)$$

$$[J_c + J_{eq}(i_g, i_d)]\dot{\omega}_c = sign(\omega_{sl}) \cdot T_c(x_c) - \frac{1}{i_g i_d}\Big[k_{tw}\theta_{cw} + \beta_{tw}\big(\frac{\omega_c}{i_g i_d} - \omega_w\big)\Big]$$

$$J_w\dot{\omega}_w = k_{tw}\theta_{cw} + \beta_{tw}\big(\frac{\omega_c}{i_g i_d} - \omega_w\big) - T_L(\omega_w)$$

$$\dot{\theta}_{cw} = \frac{\omega_c}{i_g i_d} - \omega_w$$

Simplified Model for Control

$$J_e\dot{\omega}_e = u_1 - sign(\omega_{sl})\cdot u_2 + \delta_e$$

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- δ_e copes with tracking error on T_e^{sp} and T_c^{sp}.

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- δ_e copes with tracking error on T_e^{sp} and T_c^{sp}.
- δ_c copes with
  1. tracking error on T_c^{sp}
  2. non explicitly represented dynamics
  3. unknown load T_L

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- Over time windows where ω_{sl} keeps a constant sign, the system is linear.

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- δ_c copes with
  1. tracking error on T_c^{sp}
  2. non explicitly represented dynamics
  3. unknown load T_L
- Over time windows where ω_{sl} keeps a constant sign, the system is linear.
- Since ω_e and ω_c are measured, observer can be designed for δ_e and δ_c

The Parameterized NMPC scheme

Model for prediction

$$J_e\dot{\omega}_e = u_1 - sign(\omega_{sl}) \cdot u_2 + \hat{\delta}_e$$
$$[J_c + J_{eq}]\dot{\omega}_c = sign(\omega_{sl}) \cdot u_2 - \hat{\delta}_c$$

The Parameterized NMPC scheme

Model for prediction

$$J_e\dot{\omega}_e = u_1 - sign(\omega_{sl}) \cdot u_2 + \hat{\delta}_e$$
$$[J_c + J_{eq}]\dot{\omega}_c = sign(\omega_{sl}) \cdot u_2 - \hat{\delta}_c$$

- Consider a prediction horizon $N_p \cdot \tau_s$

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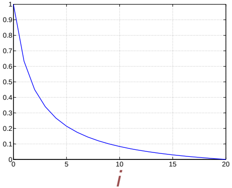
$\frac{1-i/p}{1+\lambda\cdot i/p}$, $N_p = 20$, $p = N_p$



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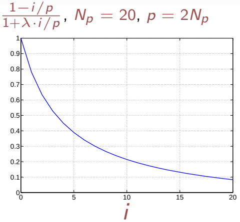

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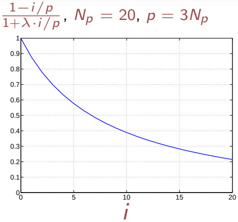

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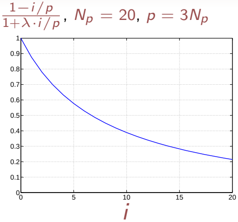

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- p monitors the speed of convergence of ω_{sl} to 0

The Parameterized NMPC scheme

Given X_{pedal}, for each choice of $p \in [N_p, \infty]$, a piecewise constant control profile can be defined according to the following unconstrained problem :

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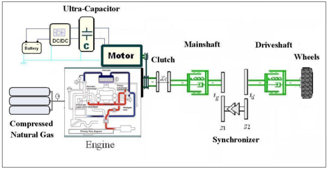
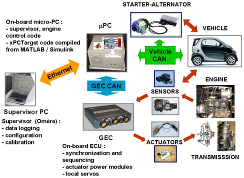
A scalar NMPC with the optimization problem :

$$\min_{p\in[N_p, N_p^{max}]} J(p,\omega) = |p - \frac{t_f(X_{pedal})}{\tau_s}| \quad \text{under saturation constraints} \quad [C(p,\omega) \leq 0]$$

The IFP's SMART demo architecture



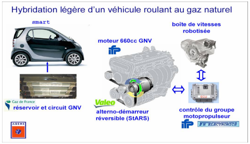
The mild-Hybrid powertrain of the VEHGAN demo-car



VEHGAN on-board Control System

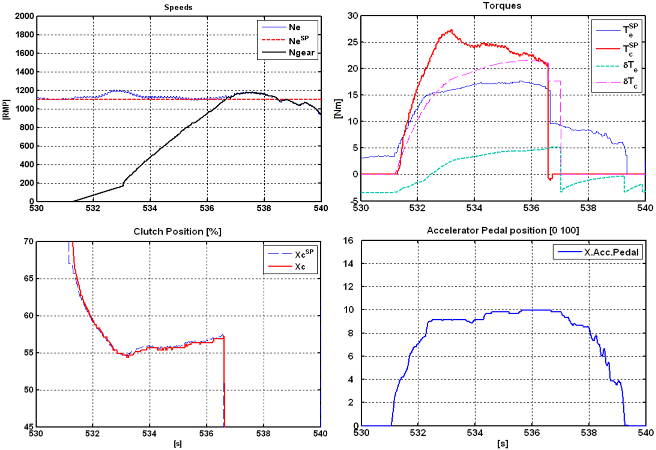
R. Amari (IFP)

P. Tona (IFP)



Demo SMART car

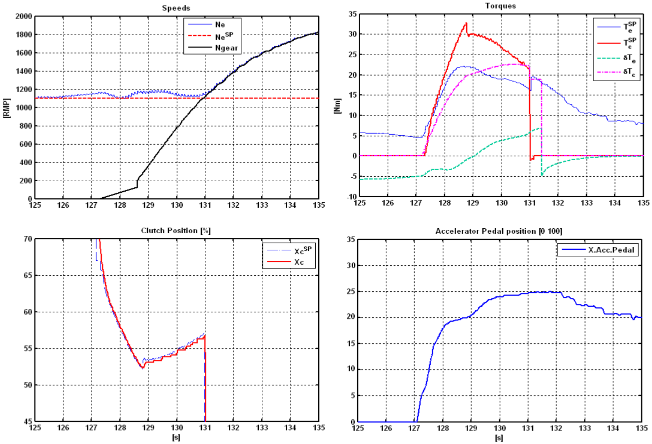
Experimental Results



Parameterized NMPC - slow start-up

(The MPC scheme controls both the engine idle speed and the start-up phase)

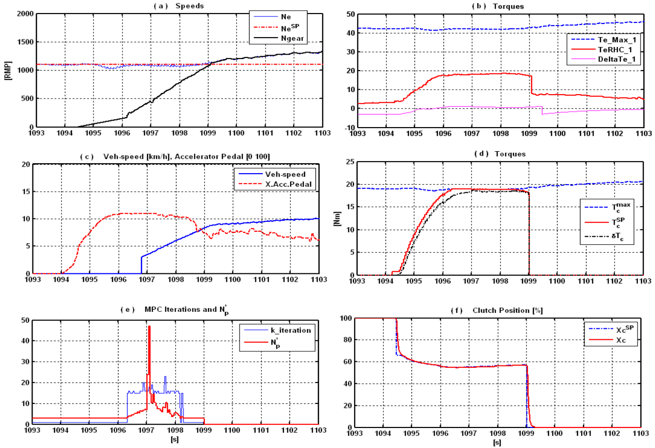
Experimental Results



Parameterized NMPC - fast start-up

(The MPC scheme controls both the engine idle speed and the start-up phase)

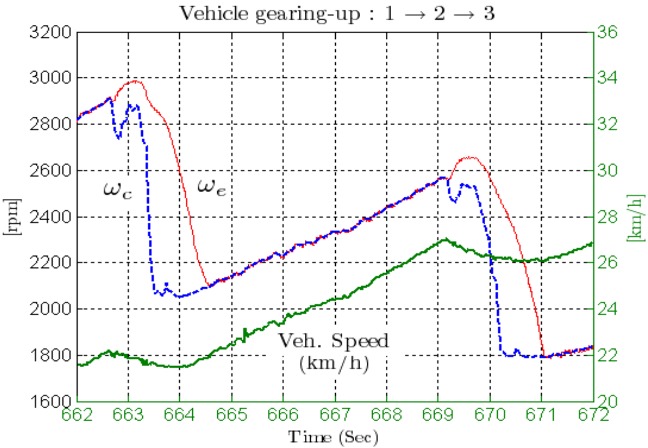
Experimental Results



Parameterized NMPC - Actuators saturation

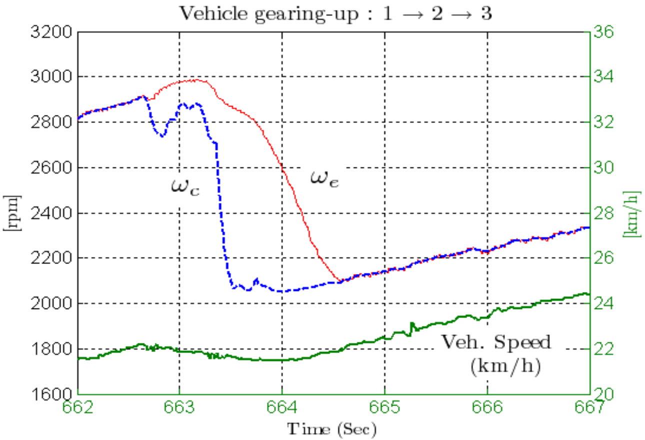
(Note the closed-loop evolution of the parameter $p =: N_p^*$)

Experimental Results



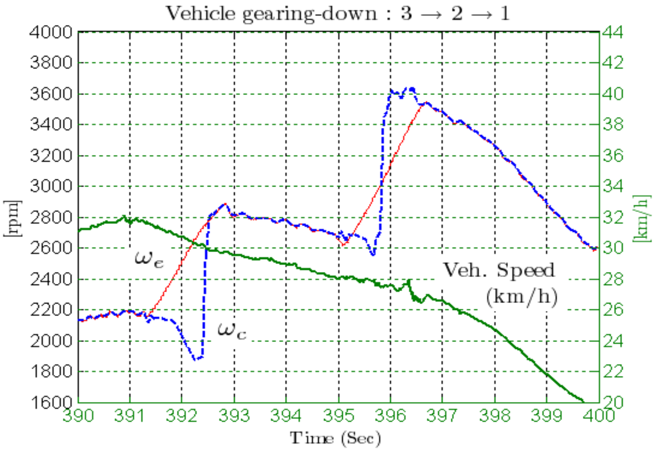
Parameterized NMPC - Gearing up

Experimental Results



Parameterized NMPC - Gearing up, 1 $\longrightarrow$ 2

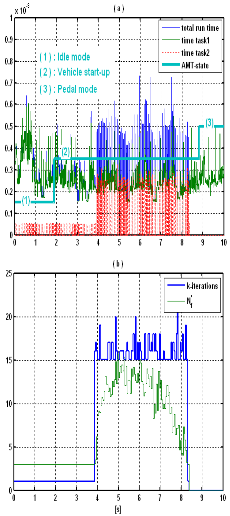
Experimental Results



Parameterized NMPC - Gearing down

Some Real-Time Implementation Issues

- Matlab/Simulink, xPC Target
- Pentium III, 256 Mb (RAM)
- Only 0.3Mb for Control Software
- Base sampling time 1 ms
- Control blocks are executed synchronously with cylinder top dead center
- variable sampling time $\in [6\ ms, 50\ ms]$
- transmission control blocks are executed at 10 ms
- Solution for p by dichotomy
- Multi-tasks config (control : task2, other task1)
- Time for preparation + 1 iteration $\approx 0.05\ ms$
- maximum (21 iterations) during saturation
- Control is computed each 50 ms
- Sampling can be drastically reduced ($< 1\ ms$)



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- Keep being engineer (at least partially)