A Novel Distributed NMPC Control Structure For Partially Cooperative Systems Under Limited Information Sharing

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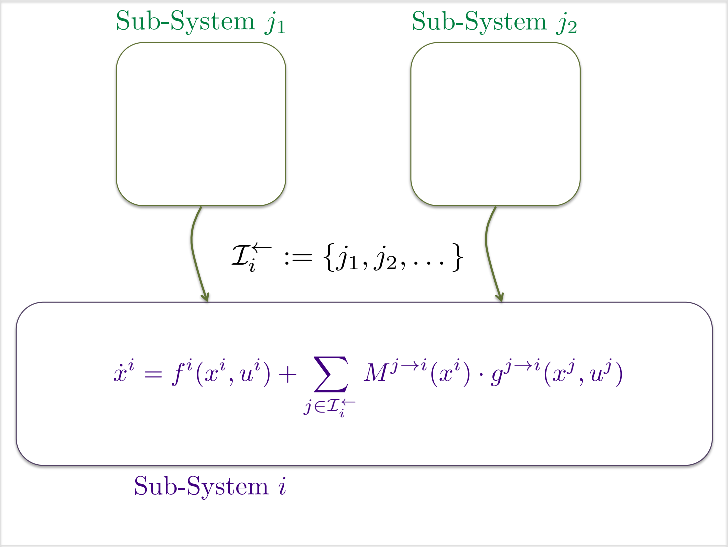
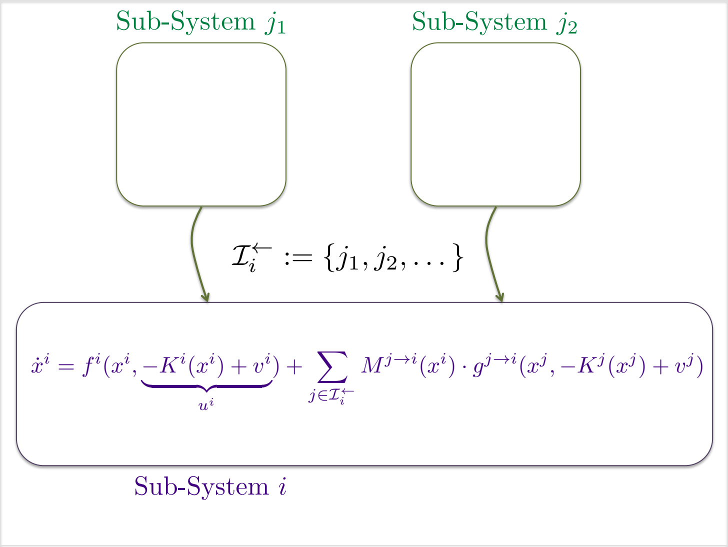
Problem Statement



Problem Statement



Problem Statement

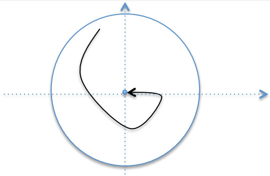
$$M^{i\to i}(x^i) = \mathbb{I}$$
$$g^{i\to i}(x^i, u^i) = f^i(x^i, u^i) - f^i(x^i, K^i(x^i))$$

$$\mathcal{I}_i^{\leftarrow} := \Big\{ i, j_1, j_2, \dots \Big\}$$

$$\dot{x}^i = f^i(x^i, K^i(x^i)) + \sum_{j\in\mathcal{I}_i^{\leftarrow}} M^{j\to i}(x^i)\cdot g^{j\to i}(x^j, u^j)$$

Sub-System i

Problem Statement



$$\dot{V}^i(x^i)\big|_{\dot{x}^i=f(x^i,K^i(x^i))} \le -W^i(x^i)$$

$$\dot{x}^i = f^i(x^i, \underbrace{-K^i(x^i)+v^i}_{u^i}) + \sum_{j\in\mathcal{I}_i^{\leftarrow}} M^{j\to i}(x^i)\cdot g^{j\to i}(x^j,-K^j(x^j)+v^j)$$

Sub-System i

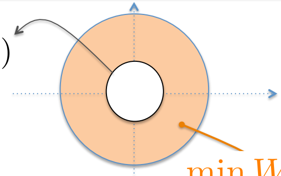
Problem Statement

$$\dot{V}^i(x^i) \leq -W^i(x^i) + \sum_{j\in\mathcal{I}_i^{\leftarrow}} L^{j\to i}(x^i) g^{j\to i}(x^j, u^j)$$

$$\dot{x}^i = f^i(x^i, \underbrace{-K^i(x^i) + v^i}_{u^i}) + \sum_{j\in\mathcal{I}_i^{\leftarrow}} M^{j\to i}(x^i)\cdot g^{j\to i}(x^j, -K^j(x^j) + v^j)$$

Sub-System i

Problem Statement



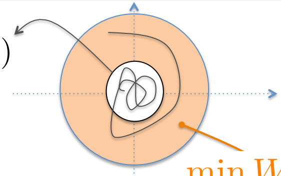
$$\dot{V}^i(x^i) \leq -W^i(x^i) + \underbrace{\sum_{j\in\mathcal{I}_i^{\leftarrow}} L^{j\to i}(x^i) g^{j\to i}(x^j, u^j)}_{\leq \gamma \cdot \bar{W}^i(\rho^i)}$$

$$\dot{x}^i = f^i(x^i, \underbrace{-K^i(x^i) + v^i}_{u^i}) + \sum_{j\in\mathcal{I}_i^{\leftarrow}} M^{j\to i}(x^i)\cdot g^{j\to i}(x^j, -K^j(x^j) + v^j)$$

Sub-System i

Problem Statement

$B^i(\rho^i)$



$\min W^i(\cdot)$

$$\dot{V}^i(x^i) \leq -W^i(x^i) + \underbrace{\sum_{j \in \mathcal{I}_i^{\leftarrow}} L^{j\to i}(x^i) g^{j\to i}(x^j, u^j)}_{\leq \gamma \cdot \bar{W}^i(\rho^i)}$$

$$\dot{x}^i = f^i(x^i, \underbrace{-K^i(x^i) + v^i}_{u^i}) + \sum_{j \in \mathcal{I}_i^{\leftarrow}} M^{j\to i}(x^i) \cdot g^{j\to i}(x^j, -K^j(x^j) + v^j)$$

Sub-System i

Main Ideas

- ρ^i defines the level cooperation of system i
- The inequality

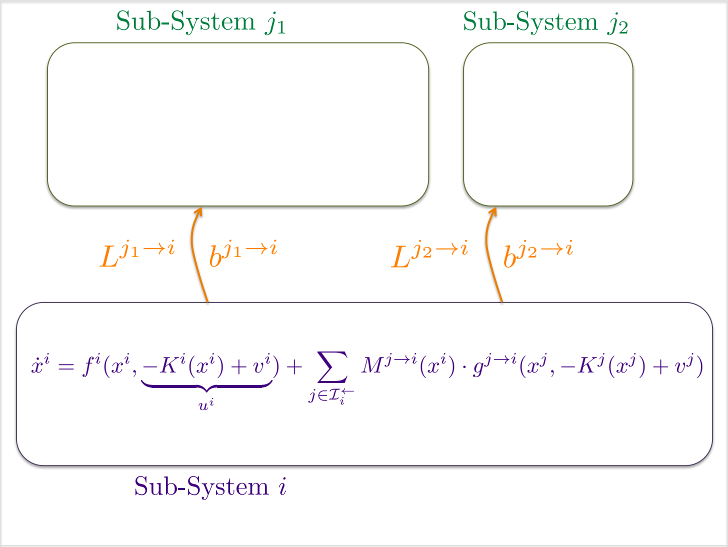
$$\underbrace{\sum_{j\in\mathcal{I}_i^{\leftarrow}} L^{j\to i}(x^i) g^{j\to i}(x^j,u^j)}_{\leq \gamma\cdot \bar{W}^i(\rho^i)}$$

is distributed into several inequalities :

$$L^{j\to i}\cdot g^{j\to i}(x^j,u^j) \leq \underbrace{d^{j\to i}\cdot \bar{W}^i(\rho^i)}_{b^{j\to i}} \quad ; \quad \Big(\sum_{j\in\mathcal{I}_i^{\leftarrow}} d^{j\to i}\leq \gamma\Big)$$

- $L^{j\to i}(x^i)$ and $b^{j\to i}$ are transmitted by i to all $j\in\mathcal{I}_i^{\leftarrow}$
- Local problems are stated and solved at each j and results are transmitted in a compact form
- ρ^i and $d^{j\to i}$ are updated accordingly.

Definition of the local problem



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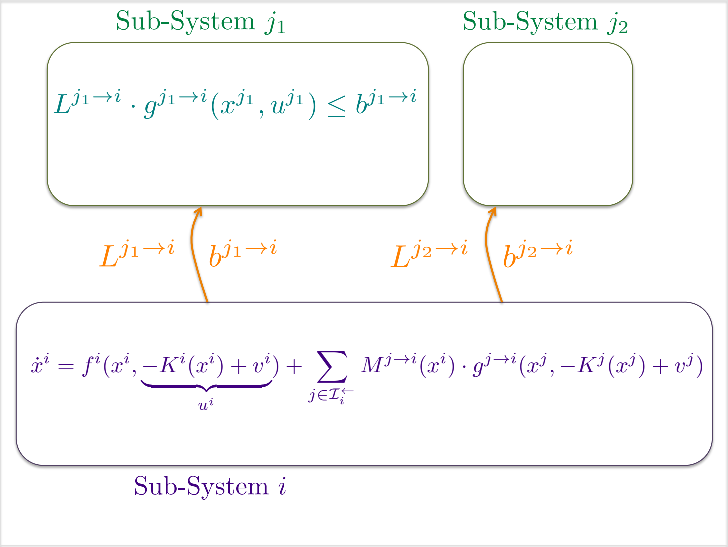

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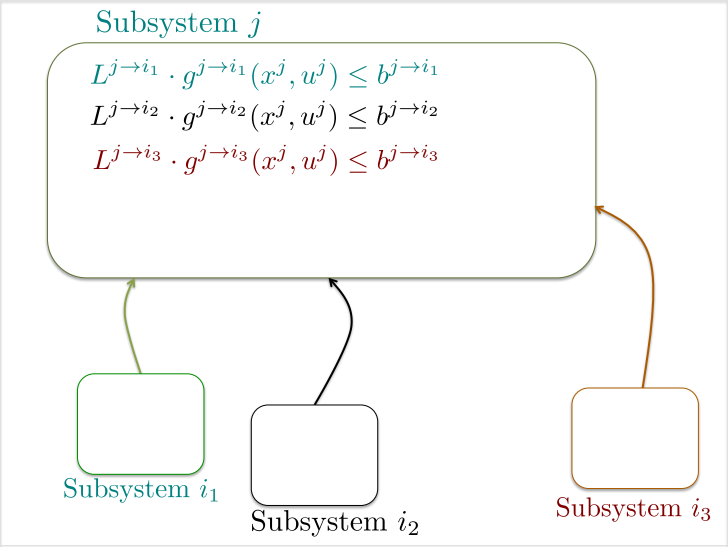

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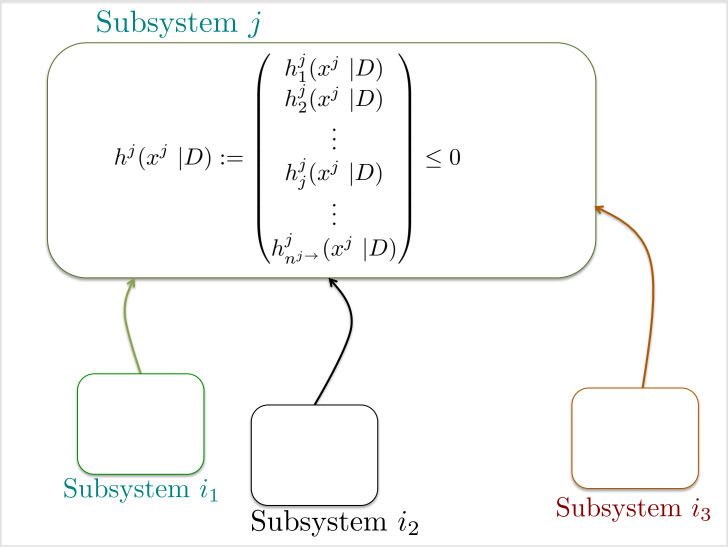


Definition of the local problem

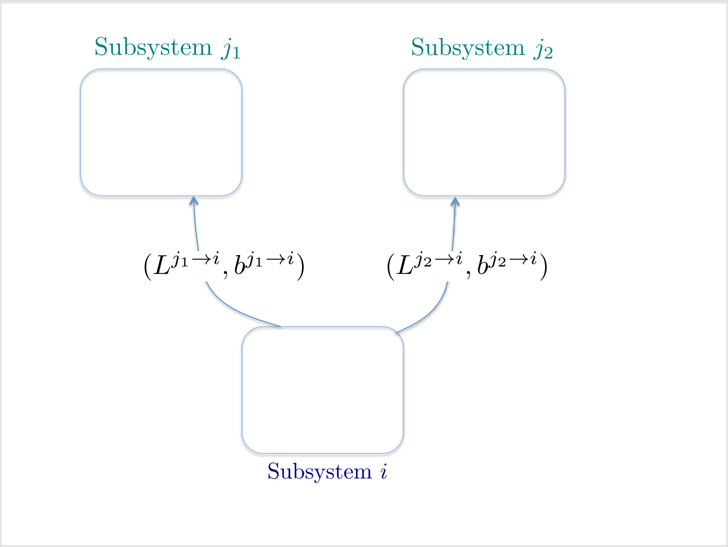
Subsystem j

NMPC Based on the Cost Function

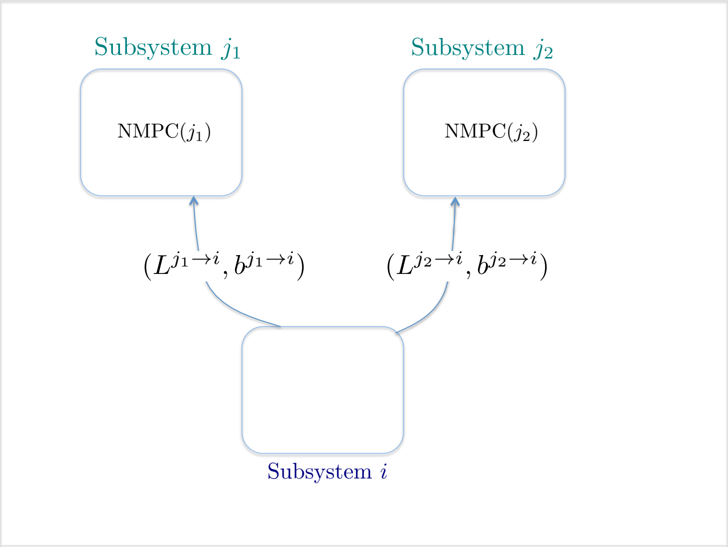
$$\min_{\tilde{v}} \sum_{k=1}^{N_p} \Big[\|v(k)\|_R^2 + \sum_{i=1}^{n^{j\rightarrow}} \max\Big\{0, \pi_i^j \cdot \hat{h}_i^j(k, \tilde{v} \mid D, \delta^j)\Big\}\Big]$$

- π_i^j defines the priorities seen by subsystem j
- $\hat{h}^j(\cdot, \dots)$ is the predicted evolution based on an updated model
- D is the data transmitted by affected neighbors $i \in \mathcal{I}^{j\rightarrow}$

Communication Protocol



Communication Protocol



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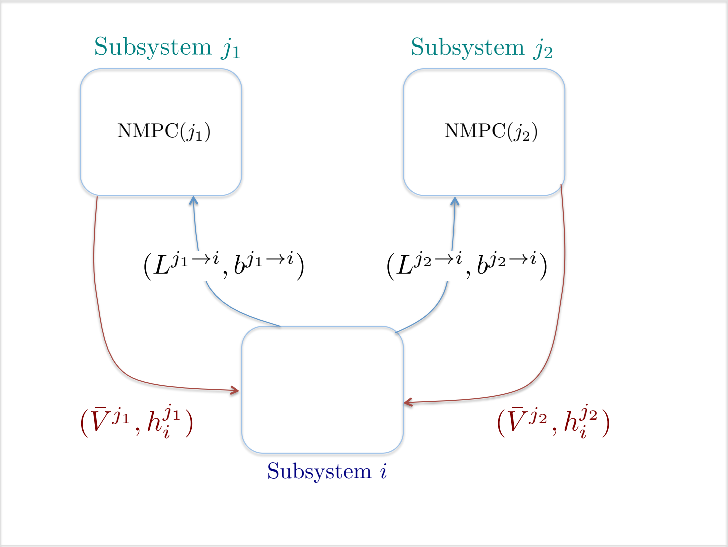

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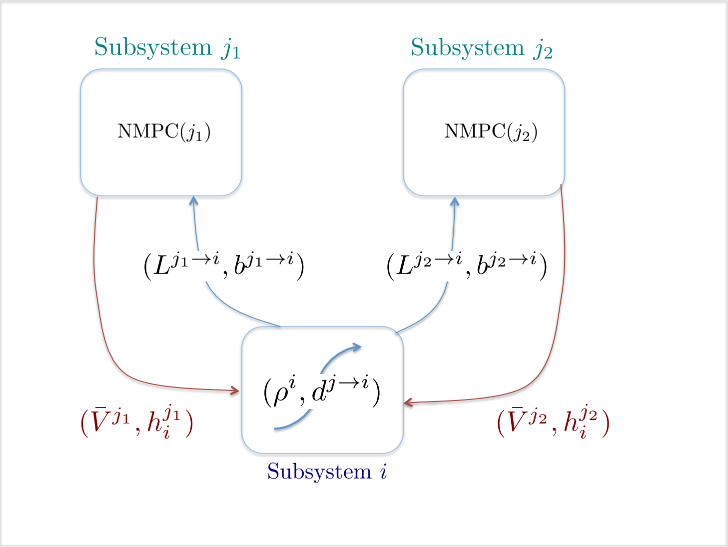


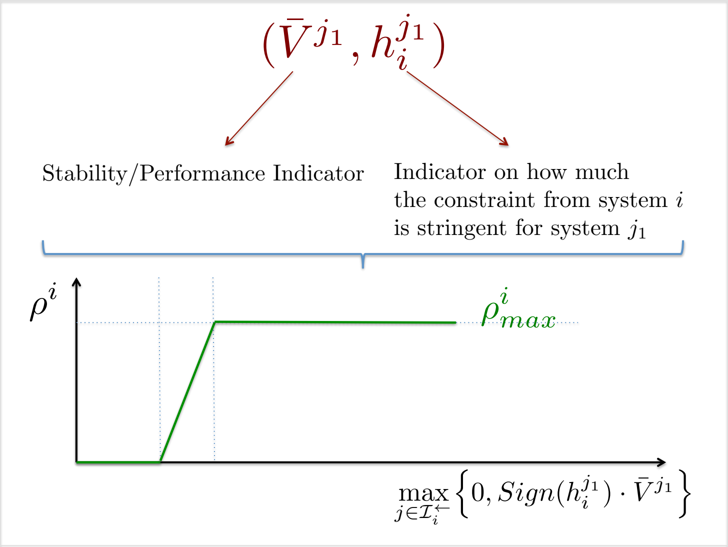
Updating the Cooperation index ρ^i

$$(\bar{V}^{j_1}, h_i^{j_1})$$

Stability/Performance Indicator

Indicator on how much the constraint from system i is stringent for system j_1

Updating the Cooperation index ρ^i



Updating the constraint distribution coefficients $d^{j\to i}$

$$d^{j\to i}(t^c_{k+1}) := \frac{\gamma}{n_{i\leftarrow}} + \alpha\Big(h^j_i(t^c_k) - \frac{1}{n_{i\leftarrow}}\sum_{\sigma\in\mathcal{I}^{\leftarrow}_i} h^\sigma_i(t^c_k)\Big)$$

Recall

The inequality

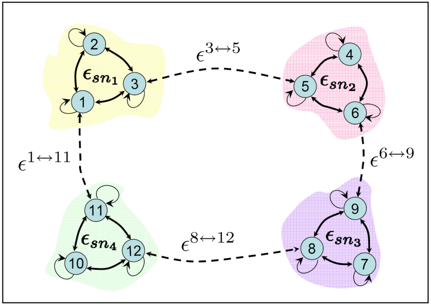
$$\underbrace{\sum_{j\in\mathcal{I}^{\leftarrow}_i} L^{j\to i}(x^i)g^{j\to i}(x^j,u^j)}_{\le \gamma\cdot\bar{W}^i(\rho^i)}$$

is distributed into several inequalities :

$$L^{j\to i}\cdot g^{j\to i}(x^j,u^j) \le \underbrace{d^{j\to i}\cdot\bar{W}^i(\rho^i)}_{b^{j\to i}} \quad ; \quad \Big(\sum_{j\in\mathcal{I}^{\leftarrow}_i} d^{j\to i}\le\gamma\Big)$$

Numerical Investigations

- A 12 subsystems network showing the benefit from cooperation
- A 3 subsystems network strengthening the key role of the priority coefficients π_i^j used in the definition of the local problems.



Some Numerical Investigations (1)

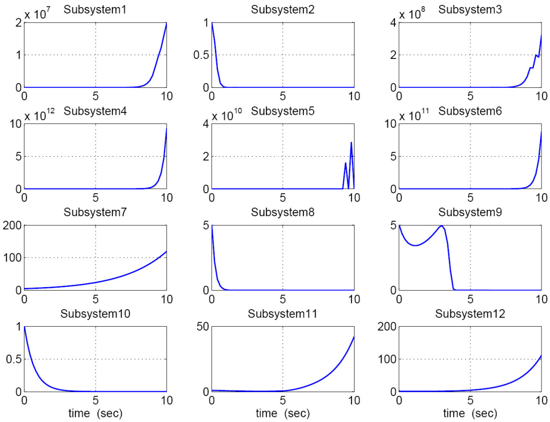


Fig. 2. Case study 1: Evolution of the network when each subsystem applies its nominally stabilizing controller. 8 subsystems diverge due to the destabilizing interconnections.

Some Numerical Investigations (2)

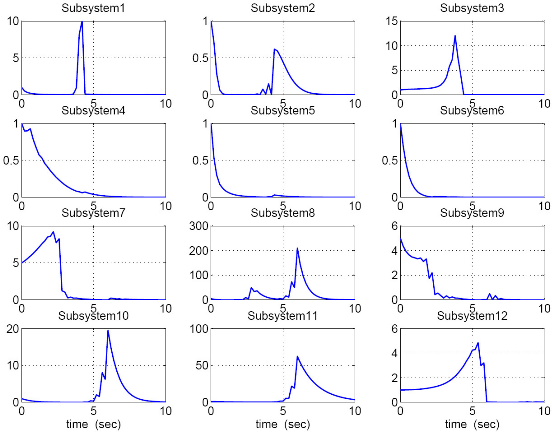


Fig. 3. Case study 1: Evolution of the network under the cooperative control scheme proposed in the present work. The cooperation stabilizes the network.

Some Numerical Investigations

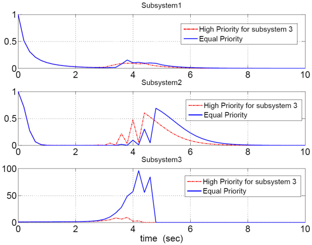


Fig. 4. Case study 2: Evolution of the closed-loop system under the cooperation scheme with two different sets of priority vectors: (Blue Solid) for $\pi^j = (1,1,1)$ for all j and (red-dotted) for $\pi^1 = \pi^2 = (1,1,10)$ while $\pi^3 = (1,1,1)$.

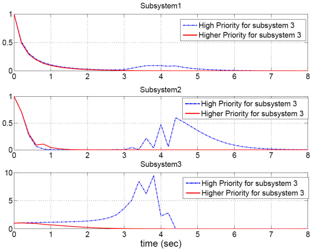


Fig. 5. Case study 2: Evolution of the closed-loop system under the cooperation scheme with two different sets of priority vectors: (Blue Solid) for $\pi^1 = \pi^2 = (1,1,10)$ (red-dotted) for $\pi^1 = \pi^2 = (1,1,100)$. In both cases, $\pi^3 = (1,1,1)$.

Conclusion & Future Work

- New heuristic for distributed partially cooperative NMPC scheme for a network of intecronnected systems with destabilizing interconnections.
- No shared knowledge on the whole state vector nor the mathematical models of subsystems
- Rather reduced amount of transmitted information
- tunable degree of cooperation and non uniform priorities assignment

Conclusion & Future Work

- For a wholly linear framework : it is possible to perform off-line computation of the framework parameters in order to enhance closed-loop stability.
- Investigating various strategies of priority assignment : Feedback, cyclic, random.
- Application to realistic & concrete case studies.