From Certification of Algorithms To Certified MPC

The missing links

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Problem Statement

Given

- A provably stable ideal MPC formulation
- A certified optimization algorithm
- A computational facility

↓

Design

RT interruptible MPC scheme so that closed-loop **certification** can be derived (if any).

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- A provably stable ideal MPC formulation
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- A provably stable ideal MPC formulation
- A certified optimization algorithm
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Co-Design

RT interruptible MPC scheme AND a hardware specification so that THE certification is achieved.

This Contribution

1. Exhibits ALL the unavoidable ingredients of such **co-design**
2. Proposes a set of sufficient conditions for MPC certification
3. Carries out exhaustive instantiation to the linear MPC case

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System

$$z_{k+1} = f(z_k, u_k, w_k)$$

z_k state
u_k control
w_k disturbance/set-point

Prediction used in MPC

$$w_{k+1} = f_d(w_k)$$

$$x := \begin{pmatrix} z \\ w \end{pmatrix}$$

Uncertain Model for MPC

$$x_{k+1} = F(x_k, u_k)$$

$$\mathbf{u}_k := \{u_k, \dots, u_{k+N_p-1}\}$$

$$\mathbf{u}_k^{(1\rightarrow i)} := \{u_k, \dots, u_{k+i-1}\}$$

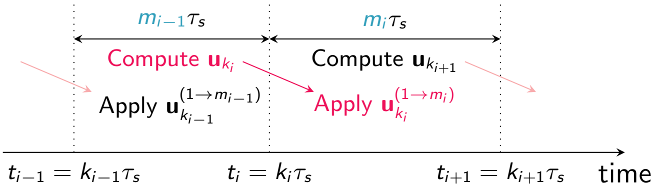
Cost Function & Constraints

$$J(\mathbf{u}, x_k) := \sum_{i=1}^{N_p} \ell(\hat{x}^{\mathbf{u}}_{k+i|k}, u_{k+i-1})$$

$$(\forall i \in I_h \cup I_s) \qquad c_i(\mathbf{u}, x_k) \leq 0$$

I_h hard constraints indices
I_s soft constraints indices

RT MPC with dynamic updating period



$\{m_i\}_{i\in\mathbb{N}}$ define successive computation periods before updating

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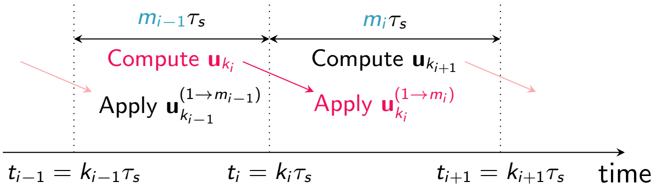

$\{m_i\}_{i\in\mathbb{N}}$ define successive computation periods before updating

Main Contribution

$$m_i = \mathcal{F}\Big(x_{k_i}, Certification, Hardware, MPC, Uncertainty, guess\Big)$$

Working Assumptions

- Certification
- Hardware
- Uncertainties
- Ideal MPC
- Extras

Certification

The algorithm $\mathcal{A}$ used to solve the OCP is s.t

$\forall(\epsilon_0, \epsilon_c) \in \mathbb{R}^2_+$, $\exists N(\epsilon_0, \epsilon_c)$ such that:

$$
\begin{aligned}
& |J(\mathbf{u}^{(i)}, x_k) - J(\mathbf{u}^{opt}(x_k), x_k)| \leq \epsilon_0 \\
(\forall i \in I_s) \quad & c_i(\mathbf{u}^{(i)}, x_k) \leq \epsilon_c \\
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for all $i \geq N(\epsilon_0, \epsilon_c)$, $\forall x_k \in \mathbb{X}$ and $\mathbf{u}^{(0)} := \mathbf{u}_k^{init}$.

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Hardware

The hardware performs a single iteration of the algorithm $\mathcal{A}$ in τ_c time units.

Working Assumptions

Certification

Hardware

Uncertainties

Ideal MPC

Extras

Uncertainties

$\exists E_1 \geq 0$ such that the prediction error satisfies:

$$\|\hat{x}^{\mathbf{u}}_{k+j|k} - x_{k+j}\| \leq E_1 \times j$$

for any x_k and $\mathbf{u}$ of interest.

Working Assumptions

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Ideal MPC

$\exists$ nonnegative function $q(\cdot)$ defined on $\mathbb{R}^n$ s.t.

$$J(\mathbf{u}^{opt}(t_{i+1}), \hat{x}(t_{i+1})) - J(\mathbf{u}^{opt}(t_i), x(t_i))$$

$$\leq$$

$$-\Delta(m_i, x(t_i)) := -\sum_{j=1}^{m_i} q(\hat{x}^{opt}_{k_i+j-1|k_i})$$

In standard formulation $m_i = 1$ and $\Delta(1, x) = \ell(x, \kappa_N(x))$ (see Mayne et al, Automatica 2000)

Working Assumptions

Certification

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Uncertainties

Ideal MPC

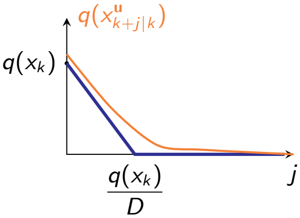
Extras

Bounded Steering Rate

$\exists D > 0$ such that for any admissible $(\mathbf{u}, x_k)$:

$$q(x^{\mathbf{u}}_{k+j|k}) \geq \max\{0, q(x_k) - D \times j\}$$

where $q(\cdot)$ is the map invoked in ideal MPC property.



Working Assumptions

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State-dependent cost and constraints

$\exists$ $K_0(\cdot)$ and $K_c(\cdot)$ s.t. for any pair $(\mathbf{u}, x^{(1)})$ and $(\mathbf{u}, x^{(2)})$ of interest:

$$|J(\mathbf{u}, x^{(1)}) - J(\mathbf{u}, x^{(2)})| \leq K_0(\|x^{(1)} - x^{(2)}\|)$$
$$\|c(\mathbf{u}, x^{(1)}) - c(\mathbf{u}, x^{(2)})\|_\infty \leq K_c(\|x^{(1)} - x^{(2)}\|)$$

Linear MPC $\rightarrow$ K_0 and K_c are computable affine bounds

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for all $i \geq N(\epsilon_0, \epsilon_c)$, $\forall x_k \in \mathbb{X}$ and $\mathbf{u}^{(0)} := \mathbf{u}_k^{init}$.

Hardware

The hardware performs a single iteration of the algorithm $\mathcal{A}$ in τ_c time units.

Uncertainties

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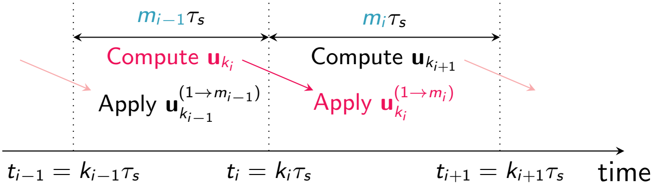
RT MPC with dynamic updating period: Change in the decision variable



$(m_i \times \tau_s)$ is the time used to iterate on $\mathcal{P}\big(\hat{x}(t_{i+1})\big)$

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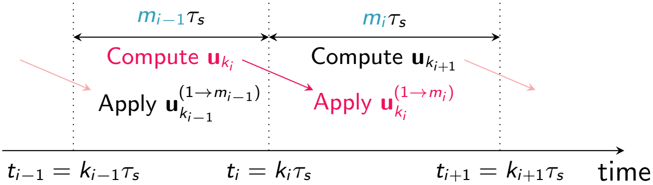

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$\Downarrow$ (thanks to algorithm certification)

Change in the decision variable $m_i \leftrightarrow \epsilon_0^{(i+1)}$

$$m_i := \bar{m}(\epsilon_0^{(i+1)}, \tau_c) := \lfloor \frac{\tau_c}{\tau_s} N(\epsilon_0^{(i+1)}, \epsilon_c) \rfloor$$

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Successive sub-optimal sequences:

$$\mathbf{u}^*_{k_{i+1}} = \mathbf{u}(t_{i+1}) := \mathcal{A}\left(\mathbf{u}^{init}_{k_{i+1}}, N(\epsilon_0^{(i+1)}, \epsilon_c)\right)$$

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$$J\big(\mathbf{u}(t_{i+1}), x(t_{i+1})\big) - J\big(\mathbf{u}(t_i), x(t_i)\big) \leq -\Phi(q(x(t_i)))?$$

$$J(\mathbf{u}(t_{i+1}), x(t_{i+1})) \quad \leq \quad J(\mathbf{u}(t_{i+1}), \hat{x}(t_{i+1})) + K_0(\|x(t_{i+1}) - \hat{x}(t_{i+1})\|)$$

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Uncertainties

$\exists E_1 \geq 0$ such that the prediction error satisfies:

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$$J(\mathbf{u}^{opt}(t_{i+1}), \hat{x}(t_{i+1})) \leq J(\mathbf{u}^{opt}(t_i), x(t_i)) - \Delta(\bar{m}(\epsilon_0^{(i+1)}, \tau_c), x(t_i))$$

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$\exists$ nonnegative function $q(\cdot)$ defined on $\mathbb{R}^n$ s.t.

$$J(\mathbf{u}^{opt}(t_{i+1}), \hat{x}(t_{i+1})) - J(\mathbf{u}^{opt}(t_i), x(t_i))$$

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RT interruptible MPC problem

How to choose $\epsilon_0^{(i+1)}$ so that:

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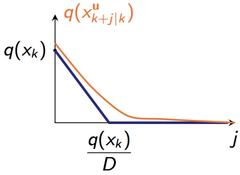

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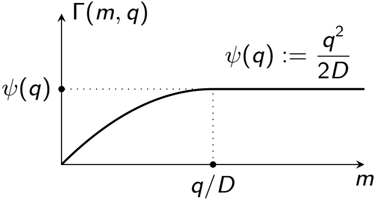

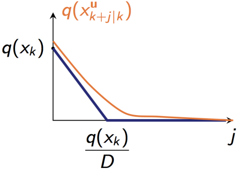

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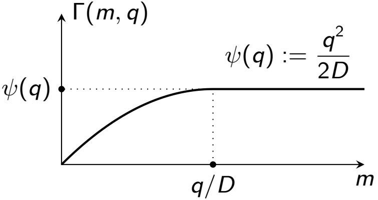

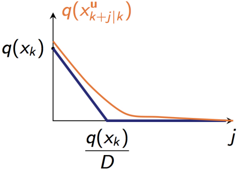

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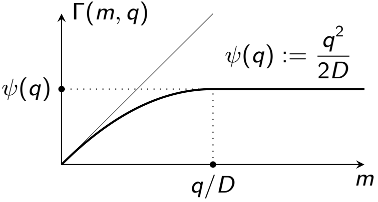

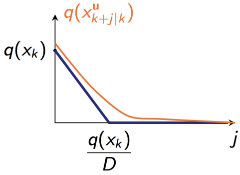

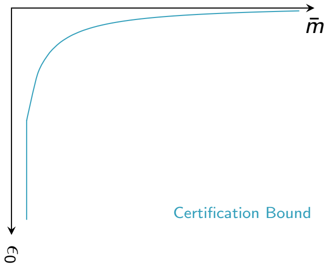

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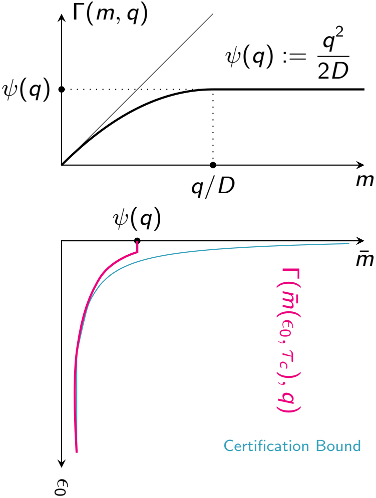

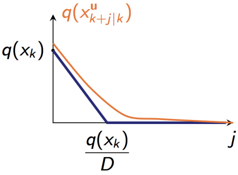

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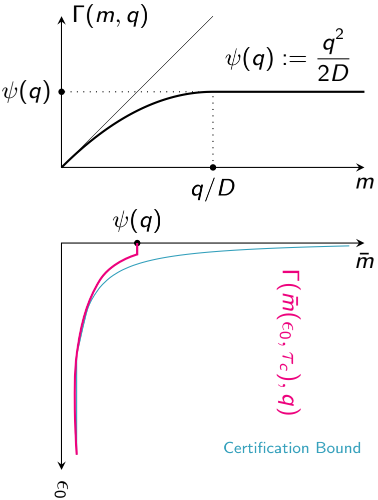
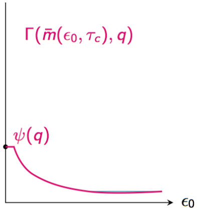
$$\Delta J \leq \epsilon_0^{(i)} + K_0(E_1 \bar{m}(\epsilon_0^{(i+1)}, \tau_c)) + \epsilon_0^{(i+1)} - \Gamma(\bar{m}(\epsilon_0^{(i+1)}, \tau_c), q(x(t_i)))$$





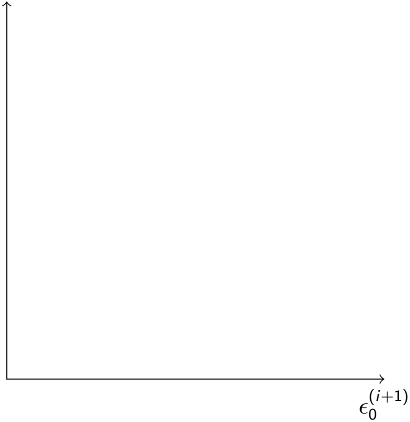
RT interruptible MPC problem

How to choose $\epsilon_0^{(i+1)}$ so that:

$$J(\mathbf{u}(t_{i+1}), x(t_{i+1})) - J(\mathbf{u}(t_i), x(t_i)) \leq -\Phi(q(x(t_i)))?$$

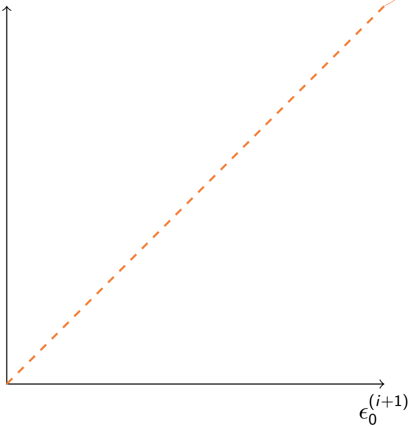
$$\Delta J \leq \epsilon_0^{(i)} + K_0\Big(E_1\bar{m}(\epsilon_0^{(i+1)},\tau_c)\Big) + \epsilon_0^{(i+1)} - \Gamma\Big(\bar{m}(\epsilon_0^{(i+1)},\tau_c), q(x(t_i))\Big)$$

$$\Delta J \leq \epsilon_0^{(i)} + K_0\left(E_1\bar{m}(\epsilon_0^{(i+1)}, \tau_c)\right) + \epsilon_0^{(i+1)} - \Gamma\left(\bar{m}(\epsilon_0^{(i+1)}, \tau_c), q(x(t_i))\right)$$

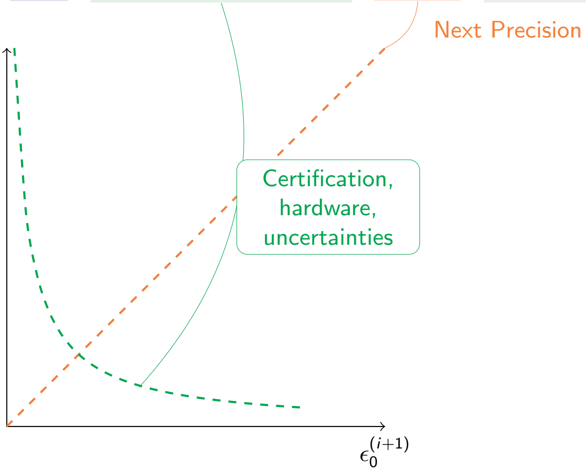


$$\Delta J \leq \epsilon_0^{(i)} + K_0\left(E_1 \bar{m}(\epsilon_0^{(i+1)}, \tau_c)\right) + \epsilon_0^{(i+1)} - \Gamma\left(\bar{m}(\epsilon_0^{(i+1)}, \tau_c), q(x(t_i))\right)$$

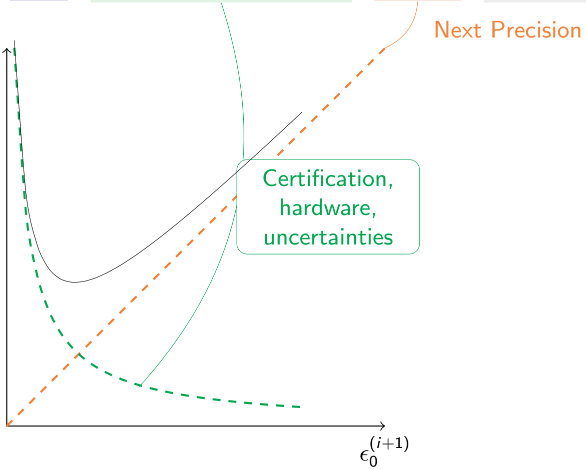
Next Precision



$$\Delta J \leq \epsilon_0^{(i)} + K_0\big(E_1\bar{m}(\epsilon_0^{(i+1)},\tau_c)\big) + \epsilon_0^{(i+1)} - \Gamma\big(\bar{m}(\epsilon_0^{(i+1)},\tau_c), q(x(t_i))\big)$$



$$\Delta J \leq \epsilon_0^{(i)} + K_0\Big(E_1\bar{m}(\epsilon_0^{(i+1)},\tau_c)\Big) + \epsilon_0^{(i+1)} - \Gamma\Big(\bar{m}(\epsilon_0^{(i+1)},\tau_c), q(x(t_i))\Big)$$



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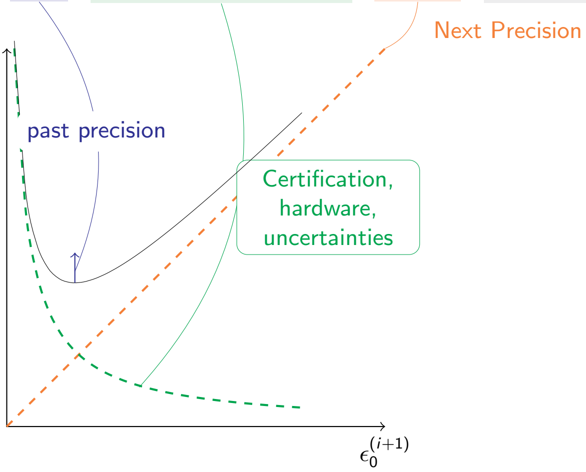

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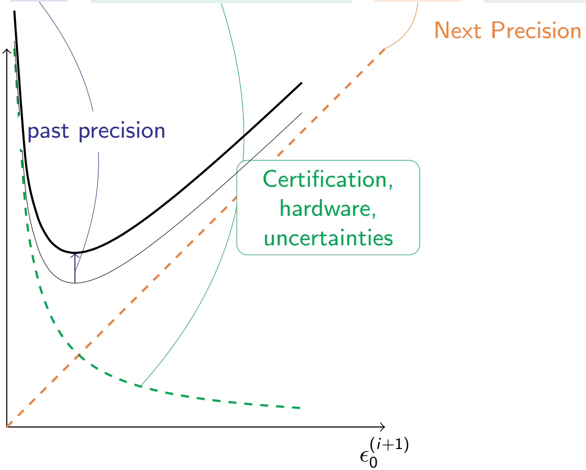

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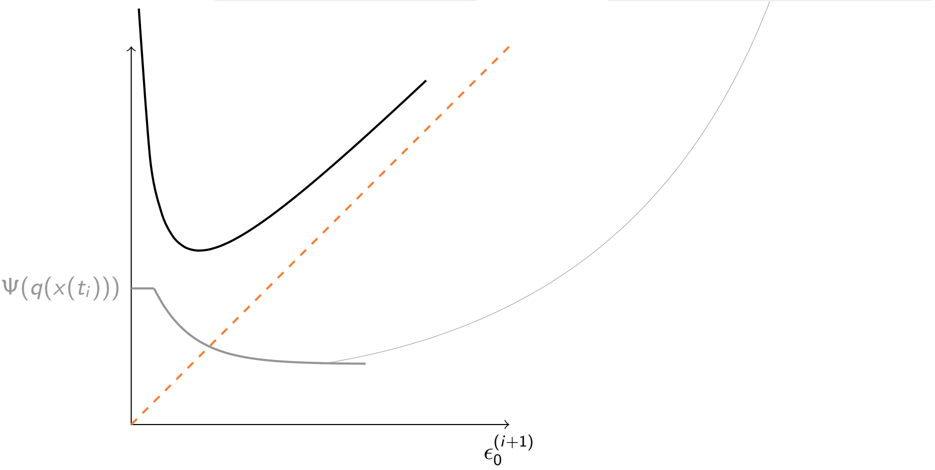

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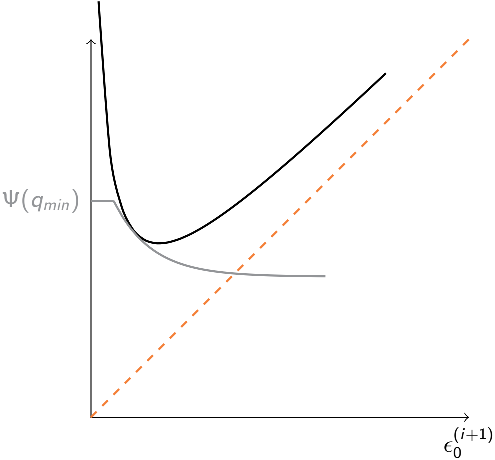

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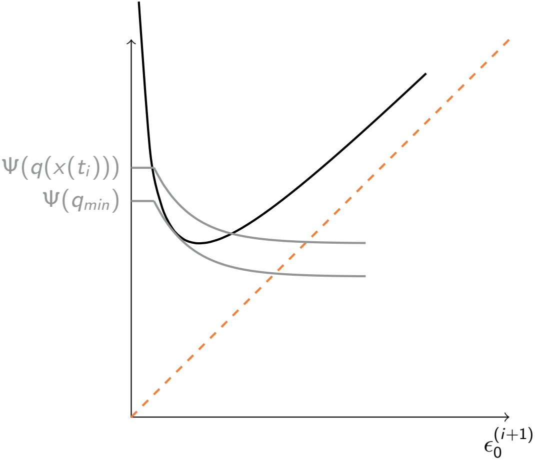

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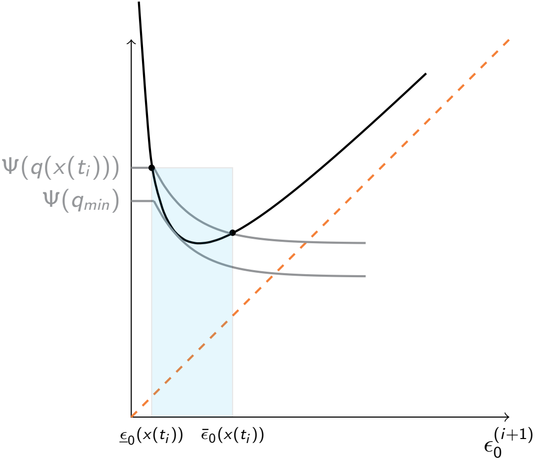

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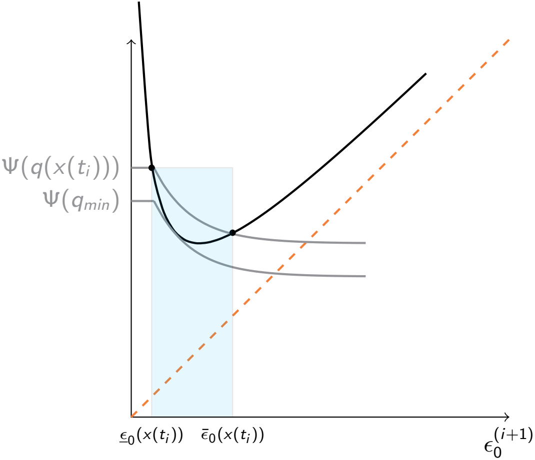
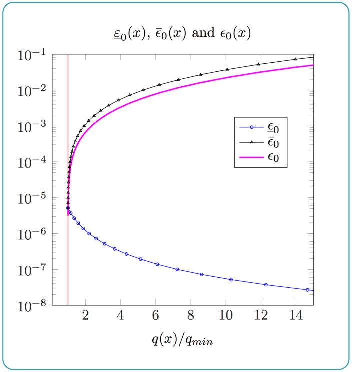
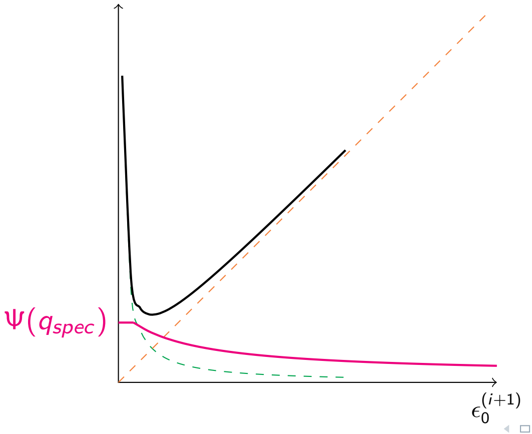


State-dependent updating period



Solution to the co-design problem

$$\Delta J \leq \epsilon_0^{(i)} + K_0\big(E_1\bar{m}(\epsilon_0^{(i+1)},\tau_c)\big) + \epsilon_0^{(i+1)} - \Gamma\big(\bar{m}(\epsilon_0^{(i+1)},\tau_c), q(x(t_i))\big)$$



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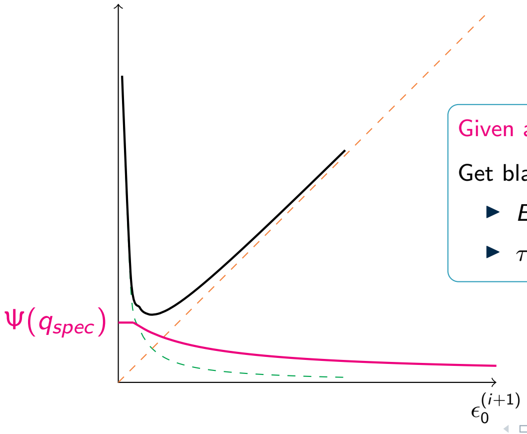


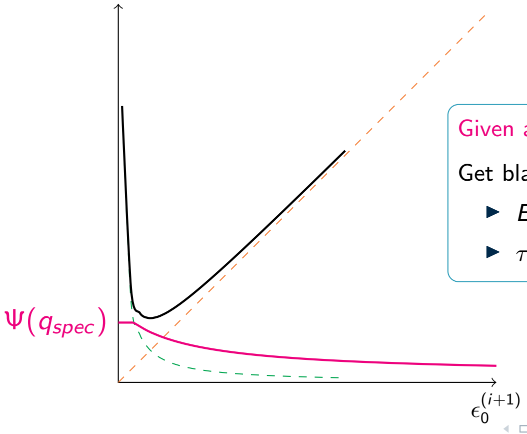
Given a specification q_{spec}

Get black curve down by:

- $E_1 \downarrow$ (ident., anticipation)
- $\tau_c \downarrow$ (hardware selection)

Solution to the co-design problem

$$\Delta J \leq \epsilon_0^{(i)} + K_0\big(E_1 \bar{m}(\epsilon_0^{(i+1)}, \tau_c)\big) + \epsilon_0^{(i+1)} - \Gamma\big(\bar{m}(\epsilon_0^{(i+1)}, \tau_c), q(x(t_i))\big)$$



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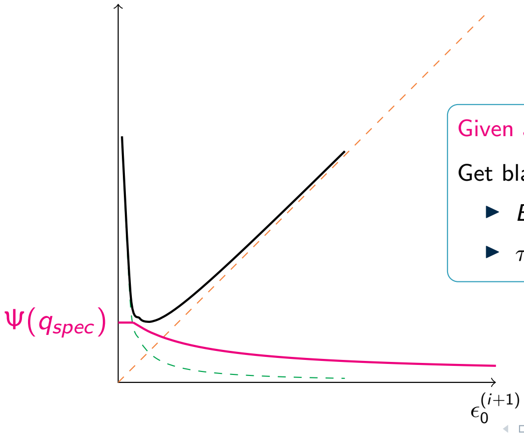

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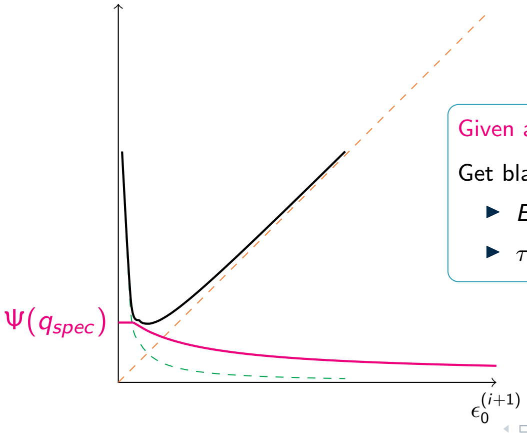

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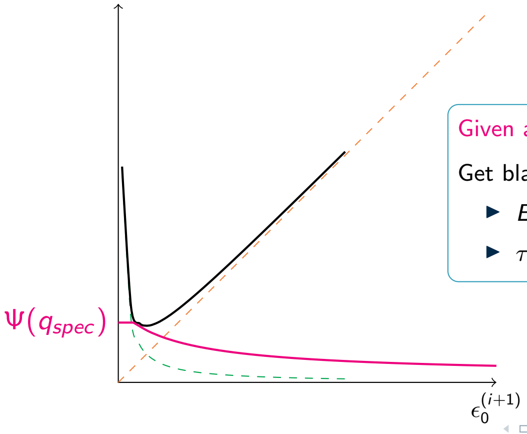

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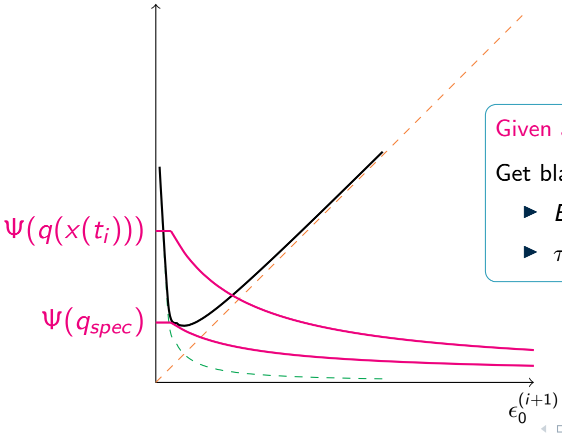

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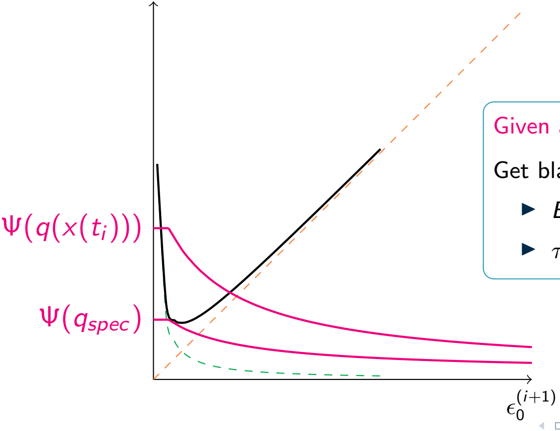

Co-Design

specification
hardware
updating strategy

Given a specification q_{spec}

Get black curve down by:

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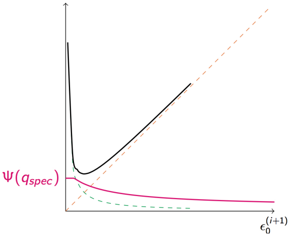
- $\tau_c \downarrow$ (hardware selection)

Achievable specification are CONSTRAINED

$$\Delta J \leq \epsilon_0^{(i)} + K_0\big(E_1\bar{m}(\epsilon_0^{(i+1)}, \tau_c)\big) + \epsilon_0^{(i+1)} - \Gamma\big(\bar{m}(\epsilon_0^{(i+1)}, \tau_c), q(x(t_i))\big)$$



Achievable specification are CONSTRAINED

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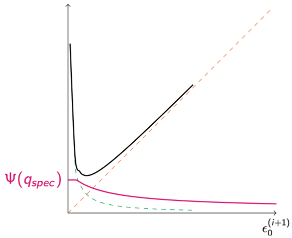

Corollary 2: We have $\|x_+ - x_+^*\| \leq \delta_{\max}$ if ϵ is chosen as

$$\epsilon \leq \frac{\mu}{2}\frac{\delta_{\max}^2}{\|B\|^2}.$$

Richter et al. 2012

Achievable specification are CONSTRAINED

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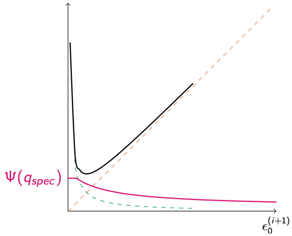

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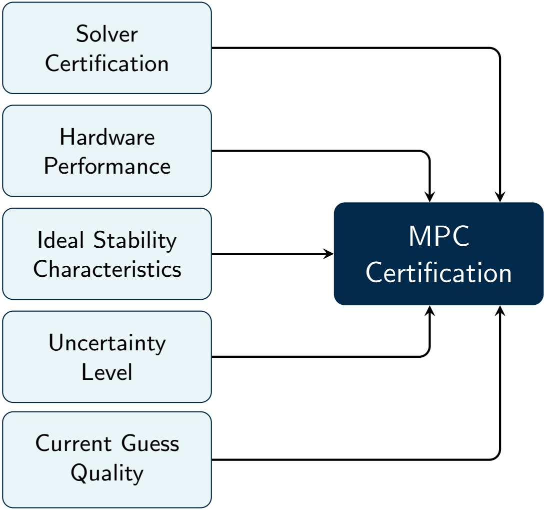
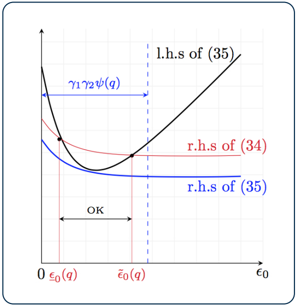
$$\epsilon \leq \frac{\mu}{2}\frac{\delta_{\max}^2}{\|B\|^2}.$$

Richter et al. 2012

Incompatible with uncertain predictions

Conclusion


Solver Certification
Hardware Performance
Ideal Stability Characteristics
Uncertainty Level
Current Guess Quality
MPC Certification


l.h.s of (35)
$\gamma_1\gamma_2\psi(q)$
r.h.s of (34)
r.h.s of (35)
OK
0
$\underline{\epsilon}_0(q)$
$\tilde{\epsilon}_0(q)$
ϵ_0

That's it . . . !

Ideal MPC

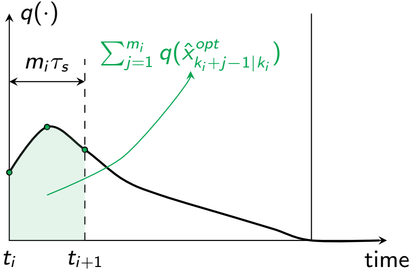
$\exists$ nonnegative function $q(\cdot)$ defined on $\mathbb{R}^n$ s.t.

$$J(\mathbf{u}^{opt}(t_{i+1}), \hat{x}(t_{i+1})) - J(\mathbf{u}^{opt}(t_i), x(t_i))$$

$$\leq$$

$$-\Delta(m_i, x(t_i)) := -\sum_{j=1}^{m_i} q(\hat{x}^{opt}_{k_i+j-1|k_i})$$

In standard formulation $m_i = 1$ and $\Delta(1, x) = \ell(x, \kappa_N(x))$ (see Mayne et al, Automatica 2000)



Case $m_i = 2$