Optimal Robust Experiment Design for Nonlinear Systems

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Optimal Robust Experiment Design: General Setting

$$x^+ = f(x, u, p)$$
$$y = h(x, p) + w$$

- $x \in \mathbb{R}^n$ state
- $u \in \mathbb{R}^m$ control
- $y \in \mathbb{R}^{n_y}$ measurement
- w measurement errors

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$\{\Omega_k(\cdot)\}_k$ contains knowledge on w

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Problem Statement

Given some known initial state x_0, compute a control profile $u(\cdot)$ defined on $[0,T]$ such that the resulting identification problem is WELL POSED

Regardless

1. The unknown value of p
2. The sequence of measurement errors $w(\cdot)$

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What would be a criterion of WELL POSDNESS for a generally non convex optimization problem ?

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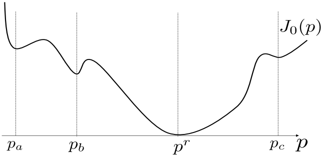

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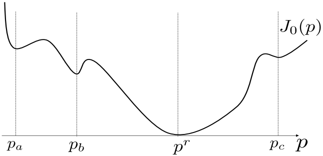

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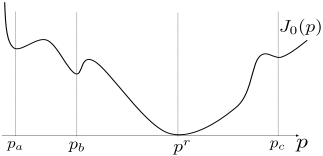

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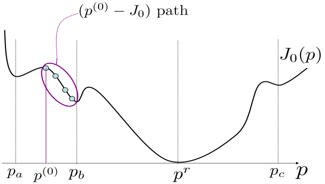

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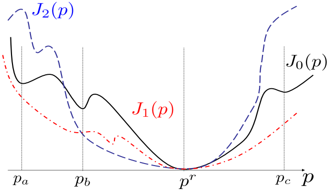

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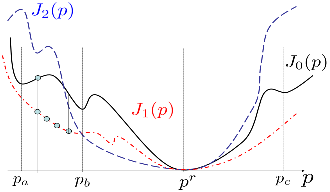

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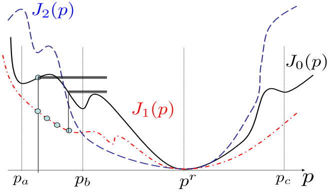

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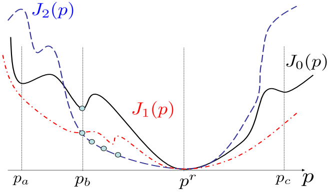

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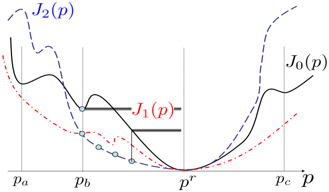

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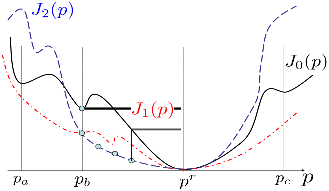

The optimization problem $\mathcal{P}_0$ is called *N-safely redundant* [Alamir, IFAC'08] iff

1) There exists a finite sequence of N cost functions J_i sharing the same global minimum $p^r \in \mathbb{P}$.

2) There exists a solver $\mathcal{S}$ and a finite integer $r^* \in \mathbb{N}$ such that:

$$\Delta_N^\gamma(p) := \min_{i\in\{0,\dots,N\}} \left[J_0(\mathcal{S}^{(r^*)}(p, J_i)) - \gamma J_0(p)\right] \leq 0$$

holds for some $\gamma \in [0,1[$ and all $p \in \mathbb{P}$

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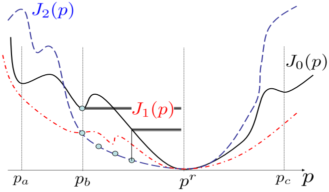

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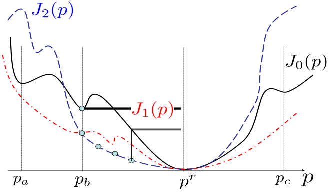

How to use this framework for nonlinear robust experiment design ?

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or

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$$d^+(\begin{pmatrix}0.8\\-2.3\end{pmatrix}, \begin{pmatrix}1\\1\end{pmatrix}) = \begin{pmatrix}0\\1.3\end{pmatrix}$$

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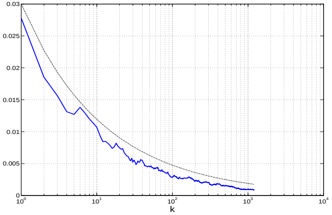

The l.h.s of the equation beside for for a MATLAB's $0.01\times$RANDN function (maximum over 1000 trials) and its approximated upper bound $\bar{w}\cdot\phi_w(k) = 0.03/(k^{0.4})$

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which is equivalent to

$$\Psi_k(q,p^r,p,w_{(.)}) := d^+\Big(\Gamma_k(q,p^r,p) - \frac{\sum_{i=0}^{k_{max}-k} w_{k+i}}{k_{max}-k+1}, \phi_w(k_{max}-k)\cdot\bar{w}\Big) = 0$$

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$$\frac{\sum_{i=0}^{k^*-1} w(k+i)}{k^*} \in \phi_w(k^*-1)\cdot[-\bar{w},\bar{w}]$$

Identification amounts to find p s.t.

$$\Gamma_k(q,p^r,p) - \frac{\sum_{i=0}^{k_{max}-k} w_{k+i}}{k_{max}-k+1} \in \phi_w(k_{max}-k)\cdot[-\bar{w},\bar{w}]$$

which is equivalent to

$$\Psi_k(q,p^r,p,w_{(.)}) := d^+\Big(\Gamma_k(q,p^r,p) - \frac{\sum_{i=0}^{k_{max}-k} w_{k+i}}{k_{max}-k+1}, \phi_w(k_{max}-k)\cdot\bar{w}\Big) = 0$$

or for all positive weighting profiles $\Phi_i : [0,T] \rightarrow \mathbb{R}_+$:

$$J_i(q,p^r,p,w_{(.)}) = \sum_{k=0}^{k_{max}} \Phi_i(k)\cdot\|\Psi_k(q,p^r,p,w_{(.)})\| = 0$$

Assumption on measurement errors w

$$\frac{\sum_{i=0}^{k^*-1} w(k+i)}{k^*} \in \phi_w(k^*-1)\cdot[-\bar{w},\bar{w}]$$

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which is equivalent to

$$\Psi_k(q,p^r,p,w_{(.)}) := d^+\left(\Gamma_k(q,p^r,p) - \frac{\sum_{i=0}^{k_{max}-k} w_{k+i}}{k_{max}-k+1}, 2\phi_w(k_{max}-k)\cdot\bar{w}\right) = 0$$

or for all positive weighting profiles $\Phi_i : [0,T] \to \mathbb{R}_+$:

$$J_i(q,p^r,p,w_{(.)}) = \sum_{k=0}^{k_{max}} \Phi_i(k)\cdot\|\Psi_k(q,p^r,p,w_{(.)})\| = 0$$

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which is equivalent to

$$\Psi_k(q,p^r,p,\quad) := d^+\Big(\Gamma_k(q,p^r,p) \qquad\qquad , 2\phi_w(k_{max}-k)\cdot\bar{w}\Big) = 0$$

or for all positive weighting profiles $\Phi_i : [0,T] \rightarrow \mathbb{R}_+$:

$$J_i(q,p^r,p,w_{(\cdot)}) = \sum_{k=0}^{k_{max}} \Phi_i(k)\cdot\|\Psi_k(q,p^r,p,w_{(\cdot)})\| = 0$$

Assumption on measurement errors w

$$\frac{\sum_{i=0}^{k^*-1} w(k+i)}{k^*} \in \phi_w(k^*-1)\cdot[-\bar{w},\bar{w}]$$

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which is equivalent to

$$\Psi_k(q,p^r,p, \quad) := d^+\Big(\Gamma_k(q,p^r,p) \qquad\qquad , 2\phi_w(k_{max}-k)\cdot\bar{w}\Big) = 0$$

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Optimal Robust Experiment Design

Find optimal excitation parameter $q_{opt}(\gamma)$ s.t.

$$q_{opt}(\gamma) = arg \min_{q \in \mathbb{Q}} \bar{\Delta}_N^{\gamma}(q) := \left[\max_{(p^r, p) \in \mathbb{P}^2} \Delta_N^{\gamma}(q, p^r, p)\right]$$

where γ is the minimum value in $[0, 1[$ such that

$$\bar{\Delta}_N^{\gamma}(q_{opt}(\gamma)) = 0$$

Consider the oscillator

$$\begin{aligned}\dot{x}_1 &= px_2 + u \\ \dot{x}_2 &= -9x_1 + (1 - x_1^2)x_2 \\ y &= x_1 + w\end{aligned}$$

$\mathbb{P} = [0.5\ ,\ 2.0]$

$\mathcal{U}(t, q) = \chi(t) \cdot q$

$q \in \mathbb{Q} := [-1.0, +1.0]^4$

$\tau = 0.0625,\ k_{max} = 60$

$x_0 = (0.1, 0)$

Initial guess $q_0 = 1 \in \mathbb{Q}$

25-nodes grid on $\mathbb{P}$

$$\begin{aligned}q_{opt}(\gamma) &= arg \min_{q \in \mathbb{Q}} \bar{\Delta}_N^\gamma(q) \\ \bar{\Delta}_N^\gamma(q) &:= \left[\max_{(p^r, p) \in \mathbb{P}^2} \Delta_N^\gamma(q, p^r, p)\right]\end{aligned}$$

gipsa-lab

Consider the oscillator

$$\begin{aligned} \dot{x}_1 &= p x_2 + u \\ \dot{x}_2 &= -9x_1 + (1 - x_1^2)x_2 \\ y &= x_1 + w \end{aligned}$$

$\mathbb{P} = [0.5\ ,\ 2.0]$

$\mathcal{U}(t,q) = \chi(t) \cdot q$

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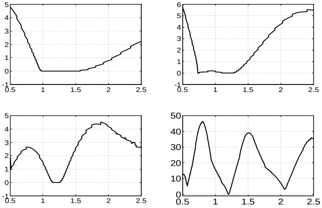
$$q_{opt}(\gamma) = arg \min_{q \in \mathbb{Q}} \bar{\Delta}_N^\gamma(q)$$

$$\bar{\Delta}_N^\gamma(q) := \left[\max_{(p^r,p) \in \mathbb{P}^2} \Delta_N^\gamma(q, p^r, p) \right]$$



Evolution of the $\neq$ cost functions $J_i(q, p^r, \cdot)$ defined by

$$\Phi_i(t) := I_{[0,30]}, I_{[0,40]}, I_{[0,50]}, I_{[0,60]}$$

for the same excitation when $p^r = 1.25$.

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Consider the oscillator

$$\begin{aligned}\dot{x}_1 &= p x_2 + u \\ \dot{x}_2 &= -9x_1 + (1 - x_1^2)x_2 \\ y &= x_1 + w\end{aligned}$$

$\mathbb{P} = [0.5\ ,\ 2.0]$

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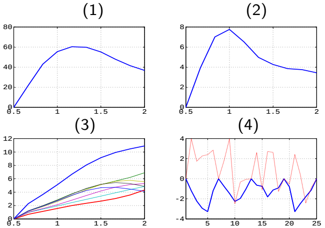

Experiments with sin signals

1. Initial worst case
2. worst case after 1-redundancy optimization
3. worst case after 8-redundancy optimization
4. Values of the $\Delta_1^{0.3}(q_{opt}^{(1)},\cdot,\cdot)$ and $\Delta_8^{0.3}(q_{opt}^{(8)},\cdot,\cdot)$

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Consider the oscillator

$$\begin{aligned}\dot{x}_1 &= p x_2 + u \\ \dot{x}_2 &= -9x_1 + (1 - x_1^2)x_2 \\ y &= x_1 + w\end{aligned}$$

$\mathbb{P} = [0.5\ ,\ 2.0]$

$\mathcal{U}(t, q) = \chi(t) \cdot q$

$q \in \mathbb{Q} := [-1.0, +1.0]^4$

$\tau = 0.0625,\ k_{max} = 60$

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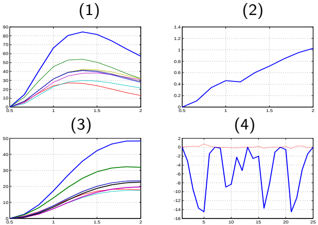
25-nodes grid on $\mathbb{P}$

$$q_{opt}(\gamma) = arg \min_{q \in \mathbb{Q}} \bar{\Delta}_N^\gamma(q)$$

$$\bar{\Delta}_N^\gamma(q) := \left[\max_{(p^r, p) \in \mathbb{P}^2} \Delta_N^\gamma(q, p^r, p)\right]$$



Experiments with sin − cos signals

1. Initial worst case
2. worst case after 1-redundancy optimization
3. worst case after 8-redundancy optimization
4. Values of the $\Delta_1^{0.3}(q_{opt}^{(1)}, \cdot, \cdot)$ and $\Delta_8^{0.3}(q_{opt}^{(8)}, \cdot, \cdot)$

gipsa-lab

Consider the oscillator

$$\begin{aligned}\dot{x}_1 &= px_2 + u \\ \dot{x}_2 &= -9x_1 + (1 - x_1^2)x_2 \\ y &= x_1 + w\end{aligned}$$

$\mathbb{P} = [0.5\ ,\ 2.0]$

$\mathcal{U}(t, q) = \chi(t) \cdot q$

$q \in \mathbb{Q} := [-1.0, +1.0]^4$

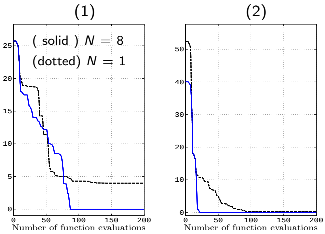
$\tau = 0.0625,\ k_{max} = 60$

$x_0 = (0.1, 0)$

Initial guess $q_0 = 1 \in \mathbb{Q}$

25-nodes grid on $\mathbb{P}$

$$\begin{aligned}q_{opt}(\gamma) &= arg \min_{q\in\mathbb{Q}} \bar{\Delta}_N^\gamma(q) \\ \bar{\Delta}_N^\gamma(q) &:= \left[\max_{(p^r,p)\in\mathbb{P}^2} \Delta_N^\gamma(q, p^r, p)\right]\end{aligned}$$



Convergence of q during the experiment optimization

1. using sin signals
2. using sin − cos signals

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Consider the oscillator

$$\begin{aligned}\dot{x}_1 &= px_2 + u \\ \dot{x}_2 &= -9x_1 + (1 - x_1^2)x_2 \\ y &= x_1 + w\end{aligned}$$

$\mathbb{P} = [0.5\ ,\ 2.0]$

$\mathcal{U}(t, q) = \chi(t) \cdot q$

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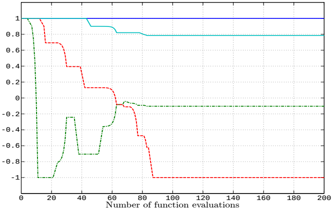
$$\bar{\Delta}_N^{\gamma}(q) := \left[\max_{(p^r,p)\in\mathbb{P}^2} \Delta_N^{\gamma}(q, p^r, p)\right]$$



Evolution of the excitation vector q during the experiment optimization for the sin experiment. (Combined SQP-Gradient with trust region-like mechanism).

Consider the oscillator

$$\begin{aligned}\dot{x}_1 &= px_2 + u \\ \dot{x}_2 &= -9x_1 + (1 - x_1^2)x_2 \\ y &= x_1 + w\end{aligned}$$

$\mathbb{P} = [0.5\ ,\ 2.0]$

$\mathcal{U}(t, q) = \chi(t) \cdot q$

$q \in \mathbb{Q} := [-1.0, +1.0]^4$

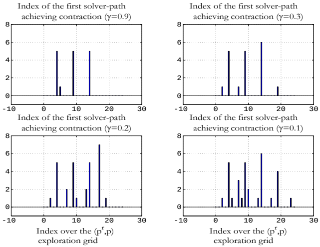
$\tau = 0.0625,\ k_{max} = 60$

$x_0 = (0.1, 0)$

Initial guess $q_0 = 1 \in \mathbb{Q}$

25-nodes grid on $\mathbb{P}$

$$\begin{aligned}q_{opt}(\gamma) &= arg \min_{q \in \mathbb{Q}} \bar{\Delta}_N^{\gamma}(q) \\ \bar{\Delta}_N^{\gamma}(q) &:= \left[\max_{(p^r, p) \in \mathbb{P}^2} \Delta_N^{\gamma}(q, p^r, p)\right]\end{aligned}$$



Index of the first solver-path achieving the contraction for different values of γ vs the index on the (p^r, p) exploration grid.

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Summarize

+ Well posdness criterion (nonlinear)
+ Account for temporal properties of noise & meas. errors
+ Robust experiment design formulation
+ Account for the specific optimization process

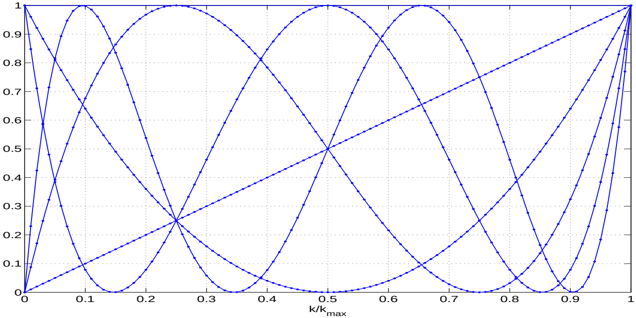
– Computationally expensive

Summarize

+ Well posdness criterion (nonlinear)
+ Account for temporal properties of noise & meas. errors
+ Robust experiment design formulation
+ Account for the specific optimization process
– Computationally expensive

Future work

- Derive a simultaneous game-like iterative algorithm
- Application to linear context
- Nonlinear Non parametric identification via graphical signatures



$$\Phi_i : \left\{1, \dots, k_{max}\right\} \rightarrow \mathbb{R}_+$$

$$\Phi_i(k) := \frac{1}{2}\Big[T_i(\frac{2k}{k_{max}} - 1) + 1\Big]$$

where the T_i's are recurrently defined by:

$$T_0(k) = 1 \ ; \ T_1(k) = k \ ; \ T_{i+1}(k) = 2kT_i(k) - T_{i-1}(k)$$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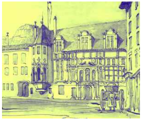
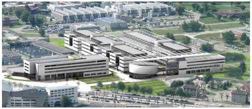
Louis Néel
(Nobel price- Physics 1970)


1788, Cradle of the French revolution


Nanotechnology


Joseph Fourier
(Grenoble admisnistrator,1801-1815)


Welcome to Grenoble


ESRF-Synchrotron


Jean-François Champollion
Studies and professorship in Grenoble

Winter Olympic Games 1968


Chartreuse liqueur distillery

IFAC NMPC-FS'06 Opening session   October 9 2006, Grenoble, France

Fast Real-Time Nonlinear-MPC

ipsa-lab

A grid-free scheme

To solve

$$\min_q \max_z C(q,z) = C_q(z) = C_z(q)$$

Use the implicit dynamics

$$q^+ = \mathcal{S}_1^{r_1}\Big(q, C_{z^*}\Big)$$
$$z^* = arg \max_{\sigma\in\{1,\dots,h\}} C_q(z_\sigma)$$
$$z_\sigma^+ = \mathcal{S}_2^{r_2}\Big(z_\sigma, C_q\Big)$$

Replace minimum value z_σ by a random trial