Monitoring Updating Period in Fast NMPC

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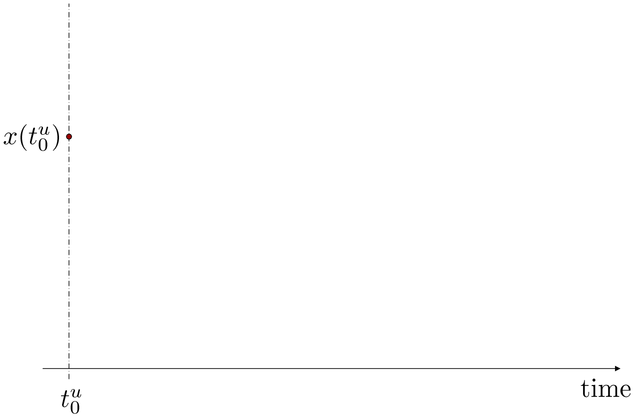
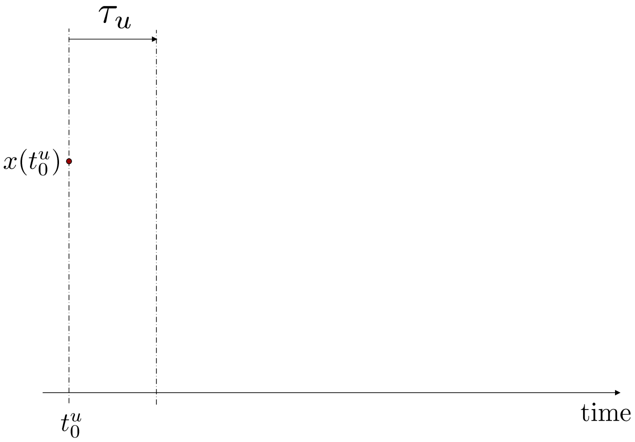
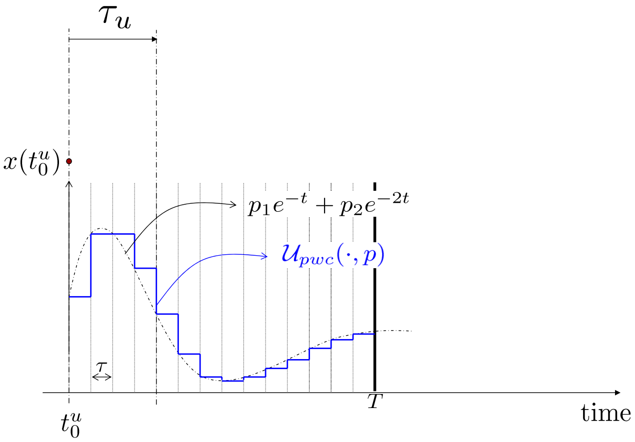
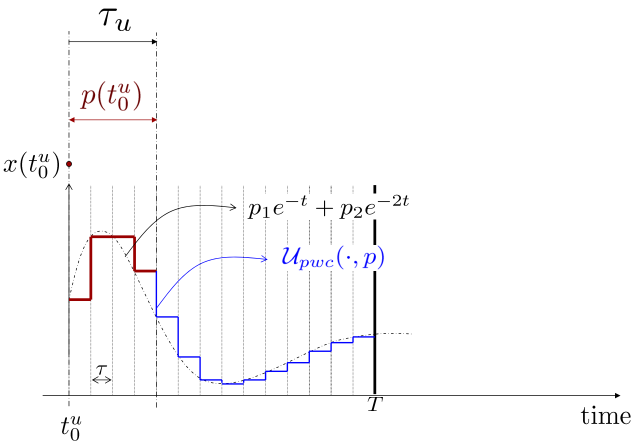
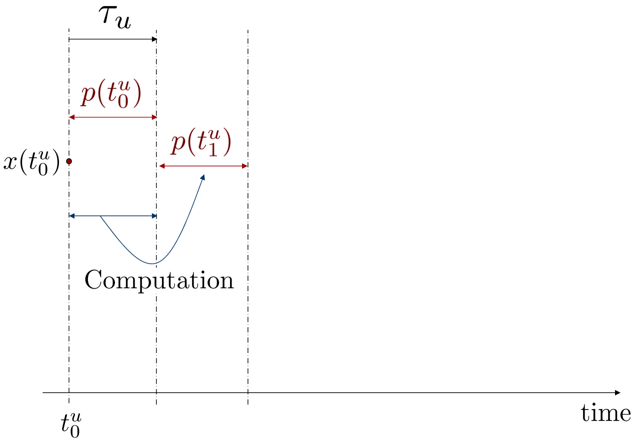
NMPC'08 - Pavia

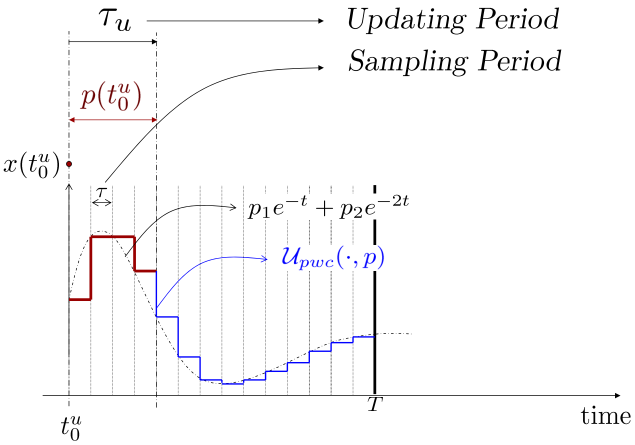

$x(t_0^u)$
t_0^u
time

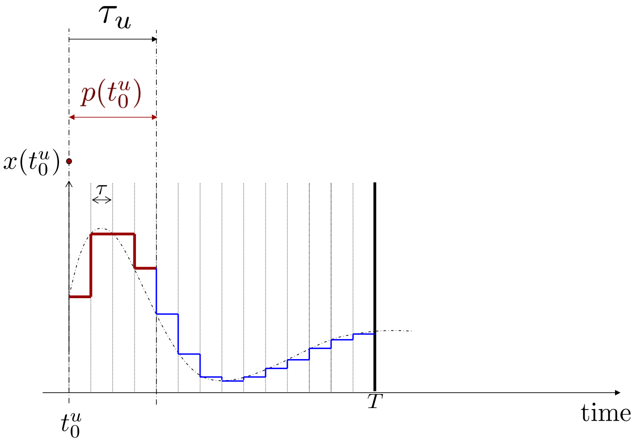

τ_u
$x(t_0^u)$
t_0^u
time

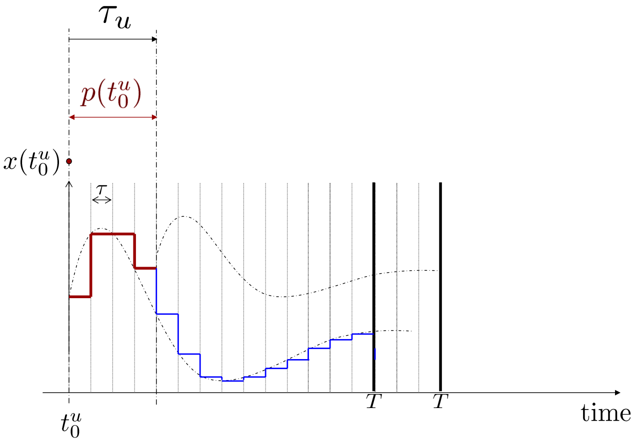

τ_u
$x(t_0^u)$
$p_1e^{-t} + p_2e^{-2t}$
$\mathcal{U}_{pwc}(\cdot, p)$
τ
T
t_0^u
time

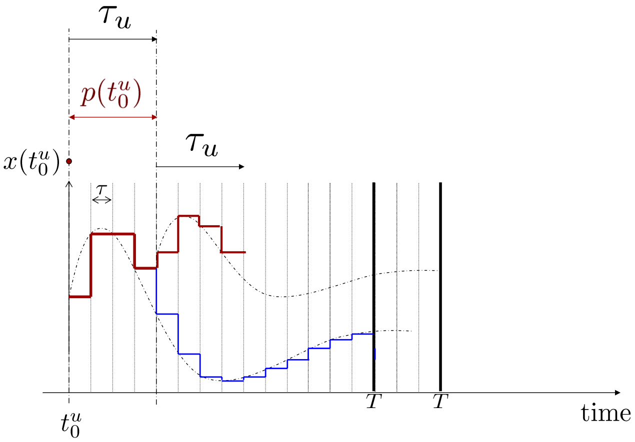

τ_u
$p(t_0^u)$
$x(t_0^u)$
$p_1 e^{-t} + p_2 e^{-2t}$
$\mathcal{U}_{pwc}(\cdot, p)$
τ
t_0^u
T
time

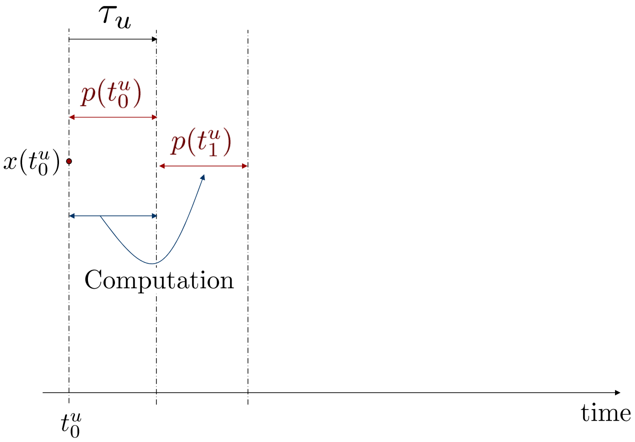

τ_u
$p(t_0^u)$
$p(t_1^u)$
$x(t_0^u)$
Computation
t_0^u
time

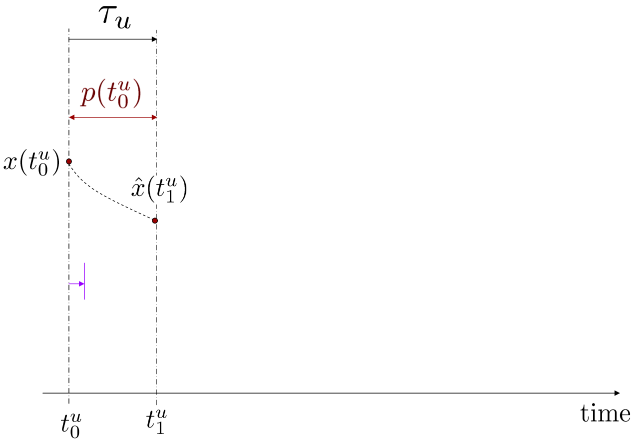

τ_u
Updating Period
Sampling Period
$p(t_0^u)$
$x(t_0^u)$
τ
$p_1e^{-t} + p_2e^{-2t}$
$\mathcal{U}_{pwc}(\cdot, p)$
T
t_0^u
time


τ_u
$p(t_0^u)$
$x(t_0^u)$
τ
t_0^u
T
time

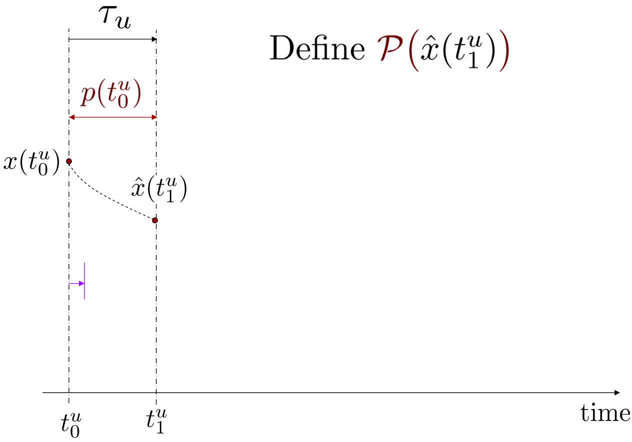

τ_u
$p(t_0^u)$
$x(t_0^u)$
τ
T
T
t_0^u
time

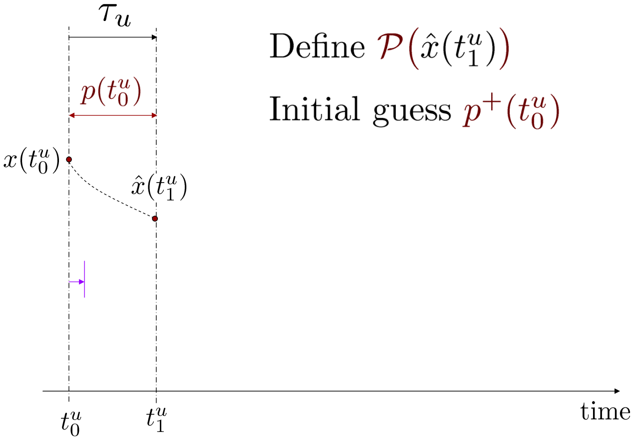

τ_u
$p(t_0^u)$
$x(t_0^u)$
τ_u
τ
T
T
t_0^u
time


τ_u
$p(t_0^u)$
$p(t_1^u)$
$x(t_0^u)$
Computation
t_0^u
time


τ_u
$p(t_0^u)$
$x(t_0^u)$
$\hat{x}(t_1^u)$
t_0^u
t_1^u
time

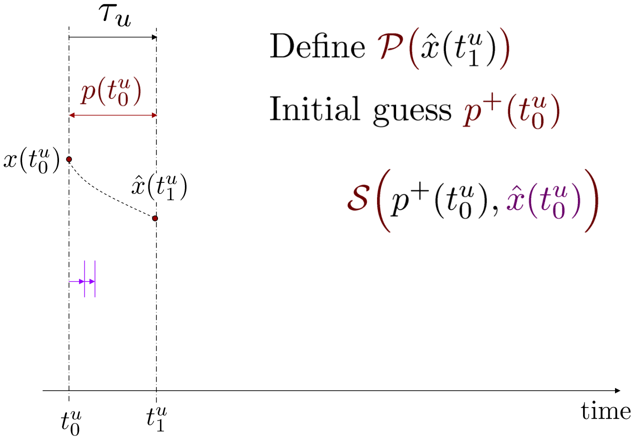
Define $\mathcal{P}\big(\hat{x}(t_1^u)\big)$





Define $\mathcal{P}\big(\hat{x}(t_1^u)\big)$

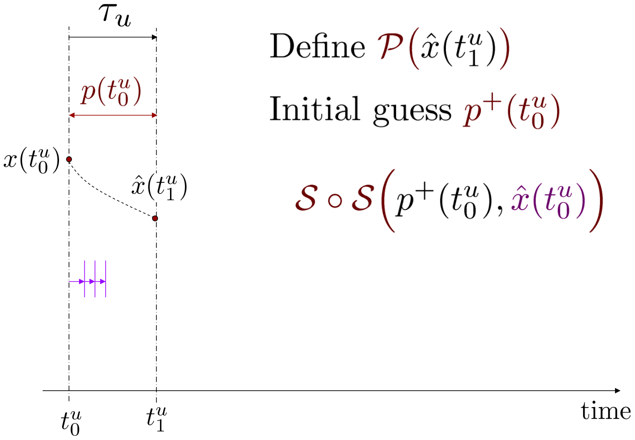
Initial guess $p^+(t_0^u)$



Define $\mathcal{P}\big(\hat{x}(t_1^u)\big)$

Initial guess $p^+(t_0^u)$

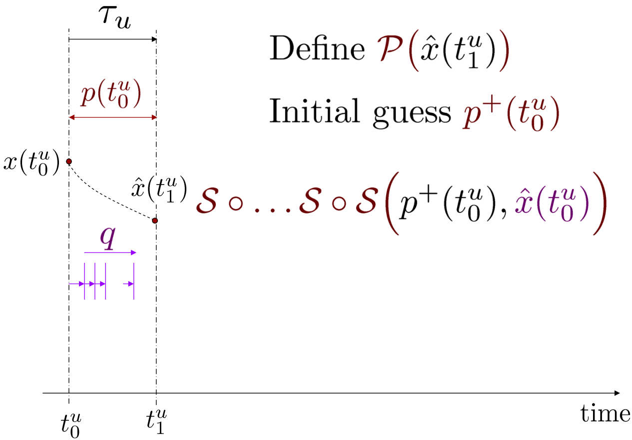
$$\mathcal{S}\Big(p^+(t_0^u), \hat{x}(t_0^u)\Big)$$



Define $\mathcal{P}\big(\hat{x}(t_1^u)\big)$

Initial guess $p^+(t_0^u)$

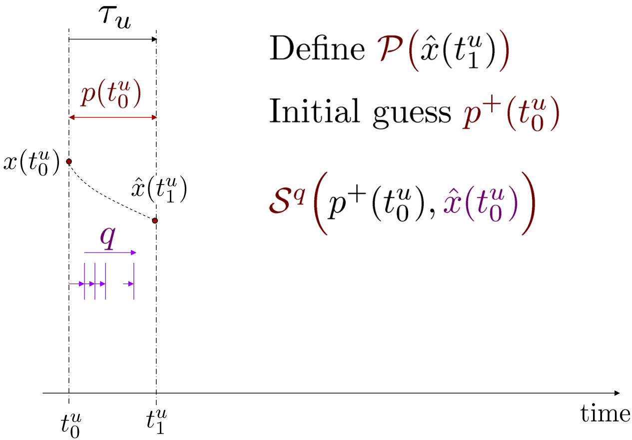
$$\mathcal{S}\circ\mathcal{S}\Big(p^+(t_0^u), \hat{x}(t_0^u)\Big)$$



Define $\mathcal{P}\big(\hat{x}(t_1^u)\big)$

Initial guess $p^+(t_0^u)$

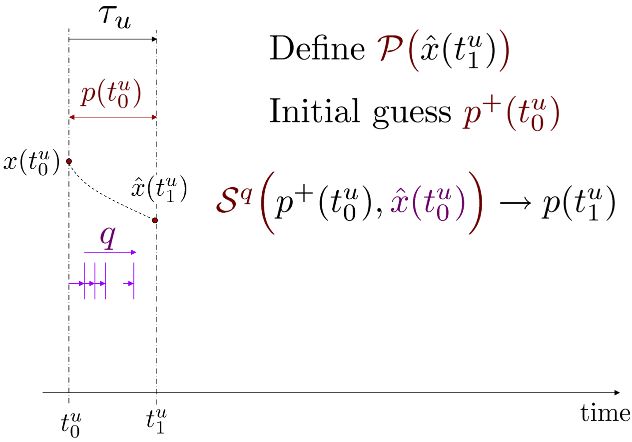
$$\mathcal{S} \circ \dots \mathcal{S} \circ \mathcal{S}\Big(p^+(t_0^u), \hat{x}(t_0^u)\Big)$$



Define $\mathcal{P}\big(\hat{x}(t_1^u)\big)$

Initial guess $p^+(t_0^u)$

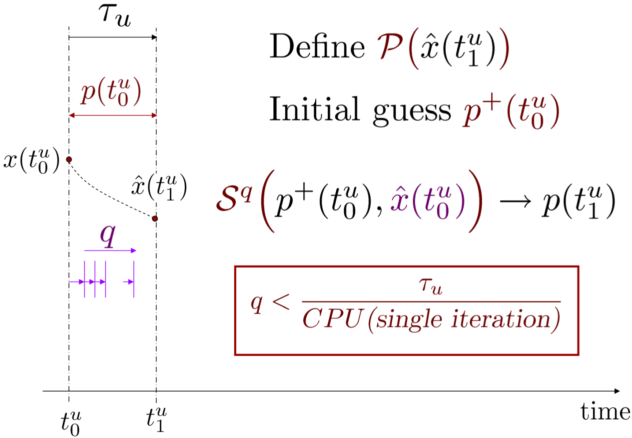
$$\mathcal{S}^q\Big(p^+(t_0^u), \hat{x}(t_0^u)\Big)$$



Define $\mathcal{P}\big(\hat{x}(t_1^u)\big)$

Initial guess $p^+(t_0^u)$

$$\mathcal{S}^q\Big(p^+(t_0^u), \hat{x}(t_0^u)\Big) \rightarrow p(t_1^u)$$

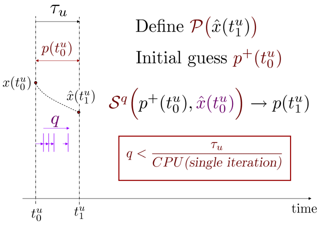


Define $\mathcal{P}\big(\hat{x}(t_1^u)\big)$

Initial guess $p^+(t_0^u)$

$$\mathcal{S}^q\Big(p^+(t_0^u), \hat{x}(t_0^u)\Big) \rightarrow p(t_1^u)$$

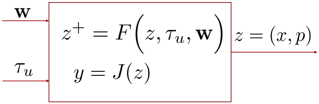
$$q < \frac{\tau_u}{CPU(\text{single iteration})}$$



This results in the extended dynamic system:

$$
\begin{aligned}
x(t_i^u) &= X^r\big(\tau_u, x(t_{i-1}^u), p(t_{i-1}^u), \mathbf{w}\big) \\
p(t_i^u) &= \mathcal{S}^q\Big(p^+(t_{i-1}^u), \underbrace{X(\tau_u, x(t_{i-1}^u), p(t_{i-1}^u))}_{\hat{x}(t_i^u)}\Big)
\end{aligned}
$$

Given $\mathcal{S}$



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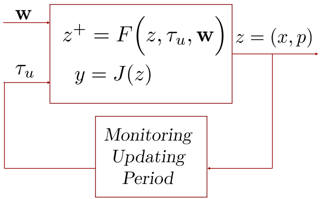
\end{aligned}
$$

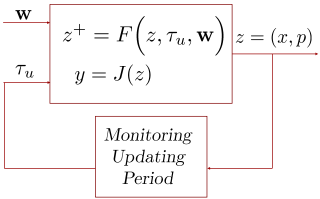
Given $\mathcal{S}$



This results in the extended dynamic system:

$$\begin{aligned} x(t_i^u) &= X^r\left(\tau_u, x(t_{i-1}^u), p(t_{i-1}^u), \mathbf{w}\right) \\ p(t_i^u) &= \mathcal{S}^q\Big(p^+(t_{i-1}^u), \underbrace{X(\tau_u, x(t_{i-1}^u), p(t_{i-1}^u))}_{\hat{x}(t_i^u)}\Big) \end{aligned}$$

Given $\mathcal{S}$



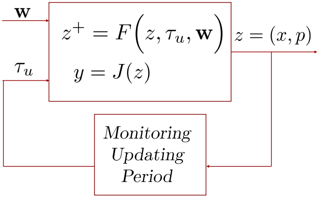
Problem Statement

Given a solver $\mathcal{S}$, propose a concrete (on-line) feedback τ_u stabilizing $y = 0$.

This results in the extended dynamic system:

$$
\begin{aligned}
x(t_i^u) &= X^r\big(\tau_u, x(t_{i-1}^u), p(t_{i-1}^u), \mathbf{w}\big) \\
p(t_i^u) &= \mathcal{S}^q\Big(p^+(t_{i-1}^u), \underbrace{X\big(\tau_u, x(t_{i-1}^u), p(t_{i-1}^u)\big)}_{\hat{x}(t_i^u)}\Big)
\end{aligned}
$$

Given $\mathcal{S}$



Problem Statement

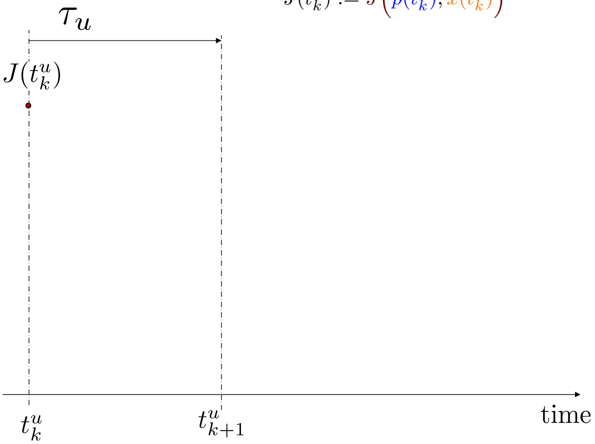
Given a solver $\mathcal{S}$, propose a concrete (on-line) feedback τ_u stabilizing $y = 0$.

NOTA: The computation time needed for this feedback must be negligible.

This results in the extended dynamic system:

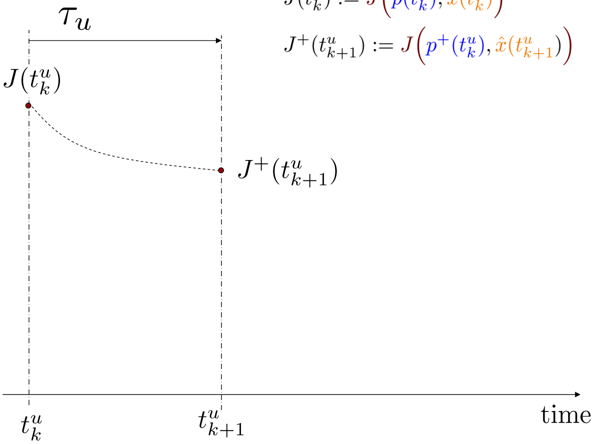
$$
\begin{aligned}
x(t_i^u) &= X^r\big(\tau_u, x(t_{i-1}^u), p(t_{i-1}^u), \mathbf{w}\big) \\
p(t_i^u) &= \mathcal{S}^q\Big(p^+(t_{i-1}^u), \underbrace{X(\tau_u, x(t_{i-1}^u), p(t_{i-1}^u))}_{\hat{x}(t_i^u)}\Big)
\end{aligned}
$$

$$J(t_k^u) := J\Big(p(t_k^u), x(t_k^u)\Big)$$



$$J(t_k^u) := J\Big(p(t_k^u), x(t_k^u)\Big)$$

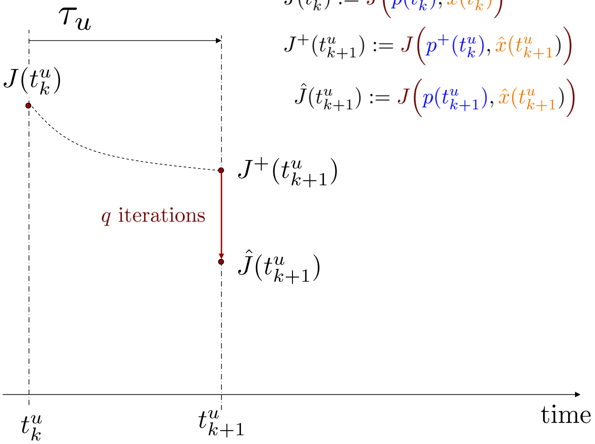
$$J^+(t_{k+1}^u) := J\Big(p^+(t_k^u), \hat{x}(t_{k+1}^u)\Big)$$

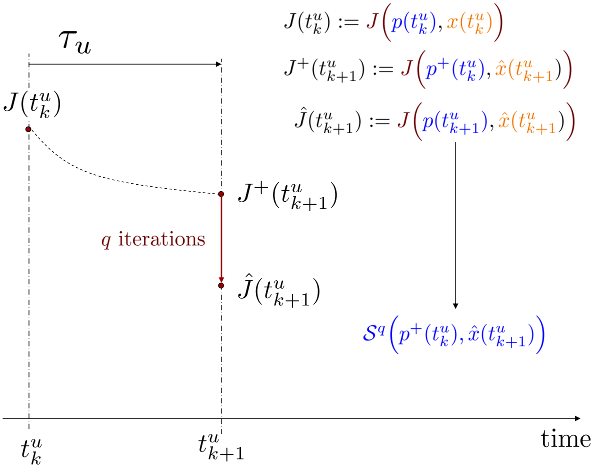


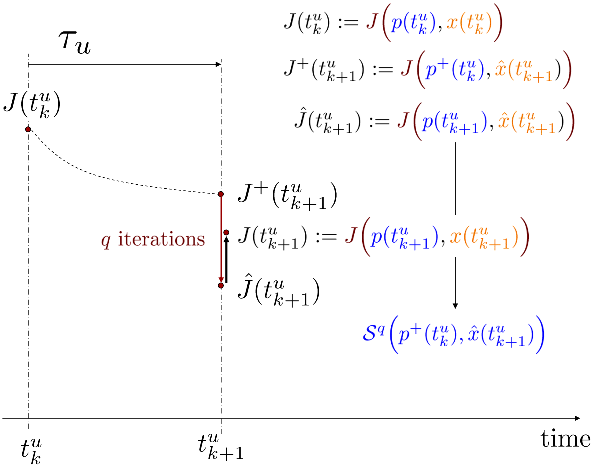
$$J(t_k^u) := J\Big(p(t_k^u), x(t_k^u)\Big)$$

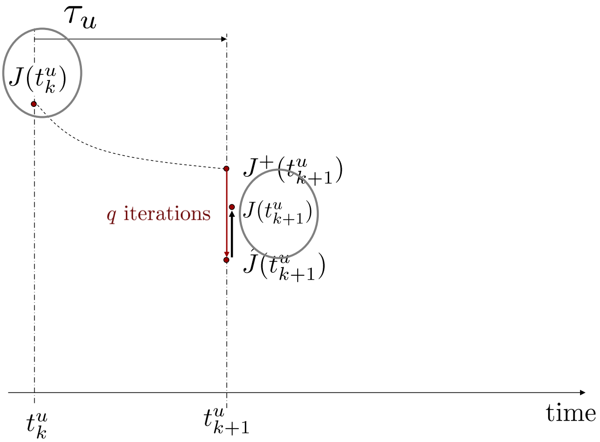
$$J^+(t_{k+1}^u) := J\Big(p^+(t_k^u), \hat{x}(t_{k+1}^u)\Big)$$

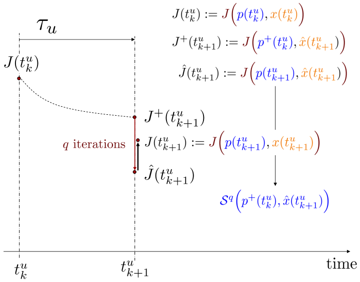
$$\hat{J}(t_{k+1}^u) := J\Big(p(t_{k+1}^u), \hat{x}(t_{k+1}^u)\Big)$$



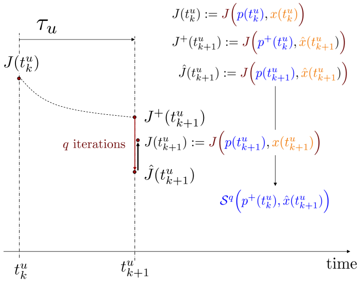

$J(t_k^u) := J\Big(p(t_k^u), x(t_k^u)\Big)$
$J^+(t_{k+1}^u) := J\Big(p^+(t_k^u), \hat{x}(t_{k+1}^u)\Big)$
$\hat{J}(t_{k+1}^u) := J\Big(p(t_{k+1}^u), \hat{x}(t_{k+1}^u)\Big)$
τ_u
$J(t_k^u)$
$J^+(t_{k+1}^u)$
q iterations
$\hat{J}(t_{k+1}^u)$
$\mathcal{S}^q\Big(p^+(t_k^u), \hat{x}(t_{k+1}^u)\Big)$
t_k^u
t_{k+1}^u
time


τ_u
$J(t_k^u)$
$J(t_k^u) := J\Big(p(t_k^u), x(t_k^u)\Big)$
$J^+(t_{k+1}^u) := J\Big(p^+(t_k^u), \hat{x}(t_{k+1}^u)\Big)$
$\hat{J}(t_{k+1}^u) := J\Big(p(t_{k+1}^u), \hat{x}(t_{k+1}^u)\Big)$
$J^+(t_{k+1}^u)$
q iterations
$J(t_{k+1}^u) := J\Big(p(t_{k+1}^u), x(t_{k+1}^u)\Big)$
$\hat{J}(t_{k+1}^u)$
$\mathcal{S}^q\Big(p^+(t_k^u), \hat{x}(t_{k+1}^u)\Big)$
t_k^u
t_{k+1}^u
time

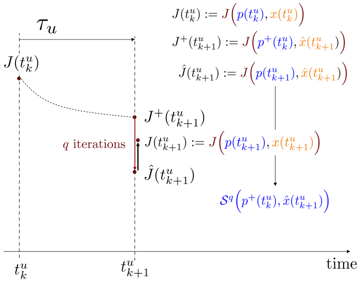

τ_u
$J(t_k^u)$
$J^+(t_{k+1}^u)$
q iterations
$J(t_{k+1}^u)$
$\acute{J}(t_{k+1}^u)$
t_k^u
t_{k+1}^u
time



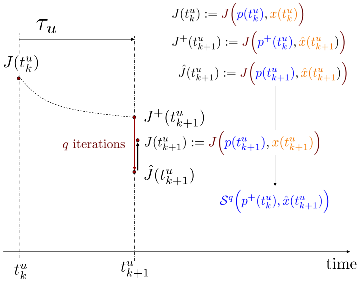
$$\frac{J(t_{k+1}^u)}{J(t_k^u)} = \left[\frac{J(t_{k+1}^u)}{\hat J(t_{k+1}^u)}\right] \times \left[\frac{\hat J(t_{k+1}^u)}{J^+(t_{k+1}^u)}\right] \times \left[\frac{J^+(t_{k+1}^u)}{J(t_k^u)}\right]$$



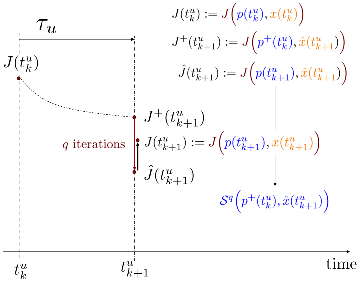
$$\frac{J(t_{k+1}^u)}{J(t_k^u)} = \left[\frac{J(t_{k+1}^u)}{\hat{J}(t_{k+1}^u)}\right] \times \underbrace{\left[\frac{\hat{J}(t_{k+1}^u)}{J^+(t_{k+1}^u)}\right]}_{E_f^{(k)}(q)} \times \left[\frac{J^+(t_{k+1}^u)}{J(t_k^u)}\right]$$



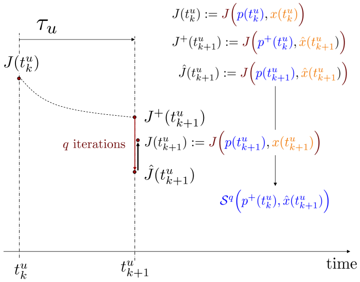
$$\frac{J(t_{k+1}^u)}{J(t_k^u)} = \underbrace{\left[\frac{J(t_{k+1}^u)}{\hat J(t_{k+1}^u)}\right]}_{\text{Uncertainty}} \times \underbrace{\left[\frac{\hat J(t_{k+1}^u)}{J^+(t_{k+1}^u)}\right]}_{E_f^{(k)}(q)} \times \left[\frac{J^+(t_{k+1}^u)}{J(t_k^u)}\right]$$



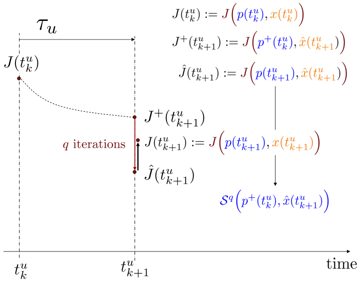
$$\frac{J(t_{k+1}^u)}{J(t_k^u)} = \underbrace{\left[\frac{J(t_{k+1}^u)}{\hat{J}(t_{k+1}^u)}\right]}_{\text{Uncertainty}} \times \underbrace{\left[\frac{\hat{J}(t_{k+1}^u)}{J^+(t_{k+1}^u)}\right]}_{E_f^{(k)}(q)} \times \underbrace{\left[\frac{J^+(t_{k+1}^u)}{J(t_k^u)}\right]}_{\text{Parametrization}}$$



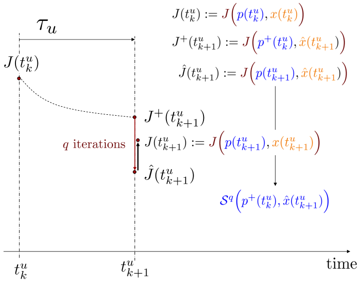
$$\frac{J(t_{k+1}^u)}{J(t_k^u)} = \underbrace{\left[\frac{\hat{J}(t_{k+1}^u)}{J^+(t_{k+1}^u)}\right]}_{E_f^{(k)}(q)} \times \underbrace{\left[\frac{J(t_{k+1}^u)}{\hat{J}(t_{k+1}^u)}\right]}_{\text{Uncertainty}} \times \underbrace{\left[\frac{J^+(t_{k+1}^u)}{J(t_k^u)}\right]}_{\text{Parametrization}}$$



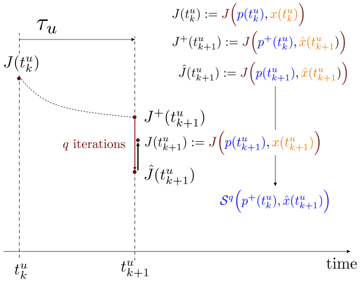
$$\frac{J(t^u_{k+1})}{J(t^u_k)} = \underbrace{\left[\frac{\hat J(t^u_{k+1})}{J^+(t^u_{k+1})}\right]}_{E_f^{(k)}(q)} \times \underbrace{\left[\frac{J(t^u_{k+1})}{\hat J(t^u_{k+1})}\right] \times \left[\frac{J^+(t^u_{k+1})}{J(t^u_k)}\right]}_{D^{(k)}(\tau_u)}$$



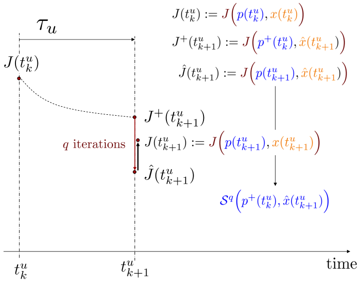
$$\frac{J(t_{k+1}^u)}{J(t_k^u)} = \underbrace{\left[\frac{\hat J(t_{k+1}^u)}{J^+(t_{k+1}^u)}\right]}_{E_f^{(k)}(int(\tau_u/\tau_c))} \times \underbrace{\left[\frac{J(t_{k+1}^u)}{\hat J(t_{k+1}^u)}\right] \times \left[\frac{J^+(t_{k+1}^u)}{J(t_k^u)}\right]}_{D^{(k)}(\tau_u)}$$



$$J(t_{k+1}^u) = \left[E_f^{(k)}(\tau_u)\right] \cdot \left[D^{(k)}(\tau_u)\right] \cdot J(t_k^u)$$



$$
\begin{aligned}
J(t_{k+1}^u) &= \Big[E_f^{(k)}(\tau_u)\Big] \cdot \Big[D^{(k)}(\tau_u)\Big] \cdot J(t_k^u) \\
&= \Big[K^{(k)}(\tau_u)\Big] \cdot J(t_k^u)
\end{aligned}
$$



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\begin{aligned}
J(t_{k+1}^u) &= \Big[E_f^{(k)}(\tau_u)\Big] \cdot \Big[D^{(k)}(\tau_u)\Big] \cdot J(t_k^u) \\
&= \Big[K^{(k)}(\tau_u)\Big] \cdot J(t_k^u)
\end{aligned}
$$

NOTA: If $K < 1$, the settling time is $t_r(\tau_u) \sim \dfrac{\tau_u}{|\log\big(K(\tau_u)\big)|}$

Updating Strategy

Provided an approximation of $K^{(k)}(\cdot)$ can be obtained, a possible updating strategy is given by:

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$$K^{(k)}(\tau_u) = \left[E_f^{(k)}(q(\tau_u))\right] \cdot \left[D^{(k)}(\tau_u)\right]$$

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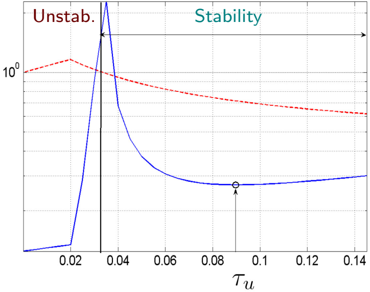
- $\alpha_f^{(k)}$ is the solution of a low dimensional linear system depending on $q_f^{(k)}$

[§ paper for details]

(a) $\alpha_f = 0.1$



- $q_f = 4$, $\tau_c = 0.005$, $\alpha_D = 8$ and $d = 1$
- (- -) $K(\tau_u)$ / (–) $t_r(\tau_u)$

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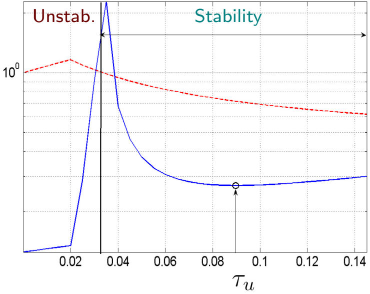

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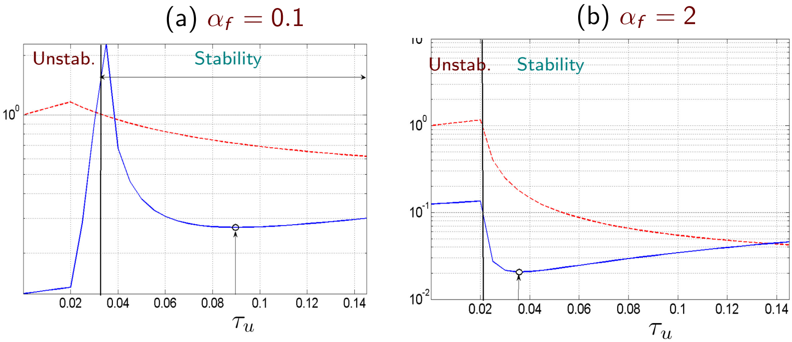
- (- -) $K(\tau_u)$ / (–) $t_r(\tau_u)$

- Instability for all $\tau_u < 0.03$
- High sensitivity of t_r over $\tau \in [0.038, 0.06]$ as $t_r \in [0.3, 0.9]$



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- Instability for all $\tau_u < 0.02$
- High sensitivity of t_r over $\tau \in [0.02, 0.03]$ as $t_r \in [0.02, 0.09]$

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$$x_1^+ = x_1 + u_1$$
$$x_j^+ = x_j + x_1^j u_2$$
$$j = 2, \dots, n$$

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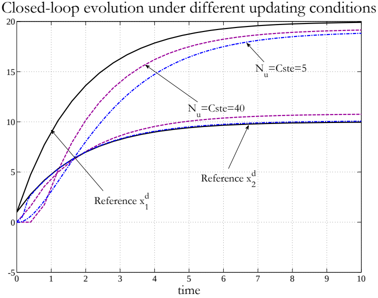

Without adaptation

- $N_u \equiv 5$
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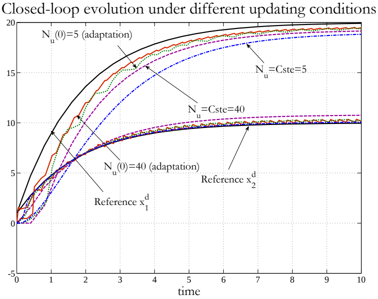

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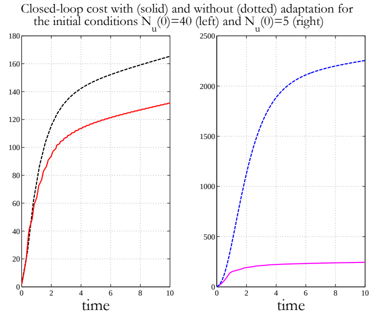

Evolution of the closed-loop cost function:

$$J_{cl}(k) := \sum_{i=0}^{k} \left[\|x(i) - x^d(i)\|^2 + \beta |p(i)| \right]$$

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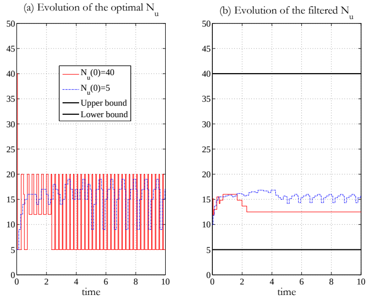

Evolution of $\tau_u(t_i^u)/\tau$ for two initialization:

$$N_u(0) = 40 \quad \text{and} \quad N_u(0) = 5$$

and their filtered values.

Conclusion & Future work

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- General Scheme (additional layer to any pre-existing solution)

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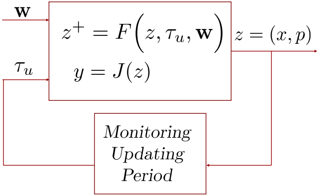
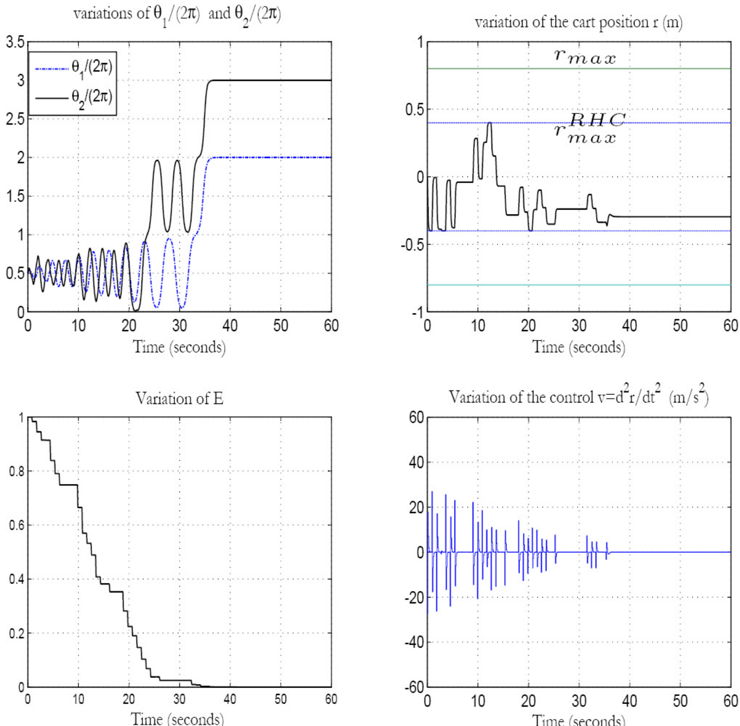
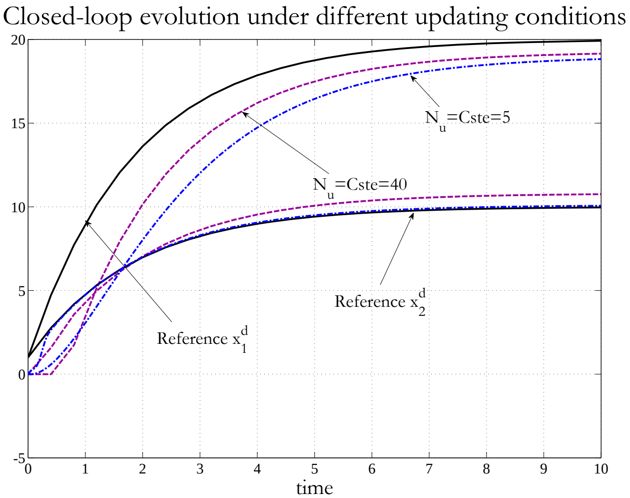
Given $\mathcal{S}$



- Investigate more elaborated model-free output feedback control design

Alamir, M. and Murilo, A. Swing-up and stabilization of a Twin-Pendulum under state and control constraints by fast NMPC scheme. Automatica, Vol. 44, pp. 1319-1324, (2008).


variations of θ1/(2π) and θ2/(2π)
θ1/(2π)
θ2/(2π)
Time (seconds)
variation of the cart position r (m)
r max
r max RHC
Time (seconds)
Variation of E
Time (seconds)
Variation of the control v=d2r/dt2 (m/s2)
Time (seconds)



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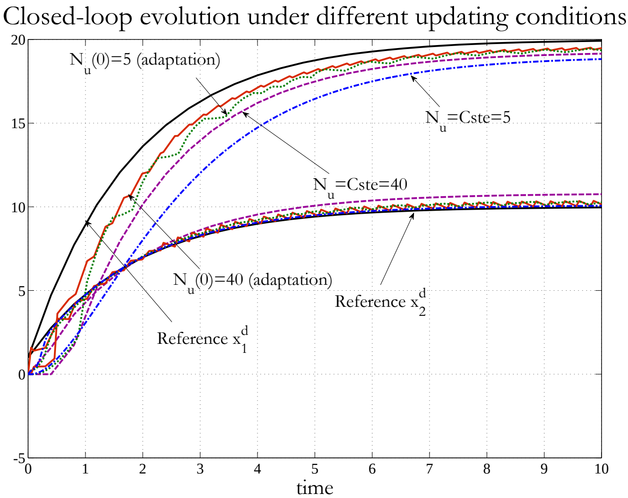

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