Fast NMPC

Some Good News and Some Facts to Keep in Mind

Mazen Alamir

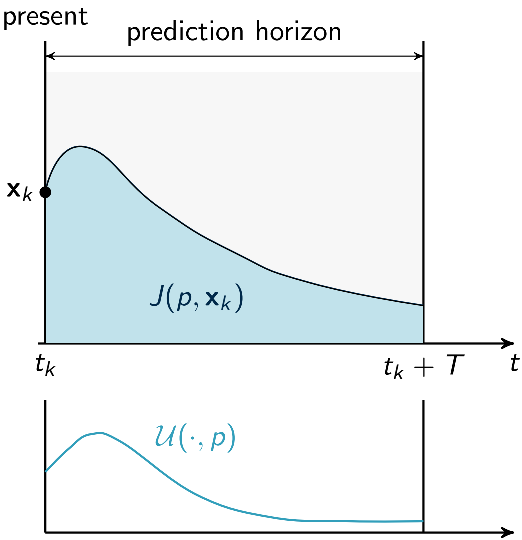
CNRS, University of Grenoble

Outline

Outline

- Ideal NMPC
- Definition of Fast NMPC
- Intuitive Sufficient Conditions

Ideal Framework: Recalls & Basic Notations



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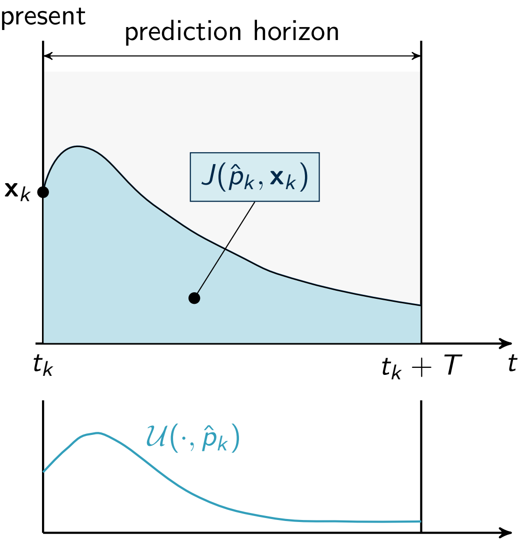

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$$\hat{p}_k = \hat{p}(\mathbf{x}_k)$$

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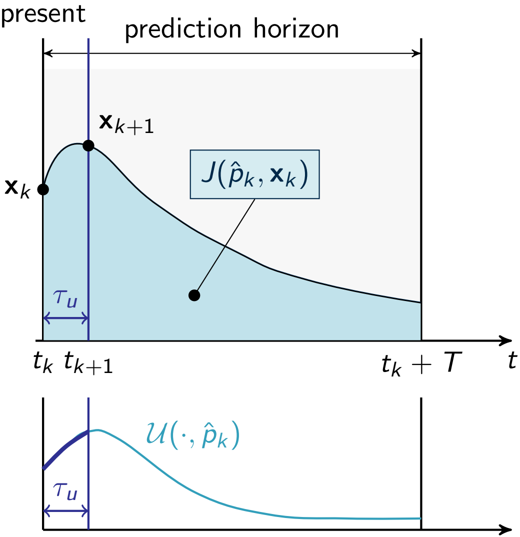

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↓

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Apply $\mathcal{U}(\cdot, \hat{p}(\mathbf{x}_k))$ during τ_u

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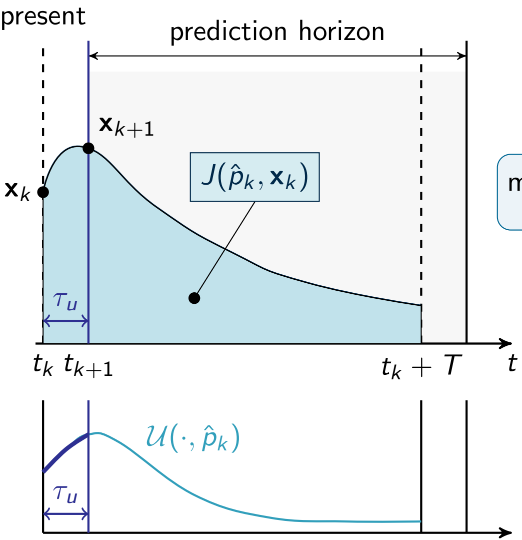

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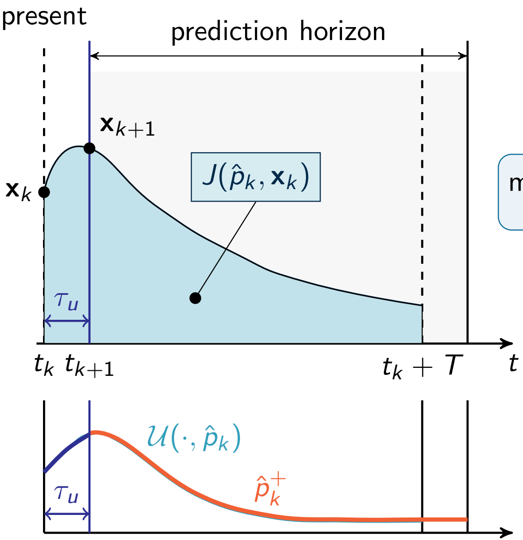

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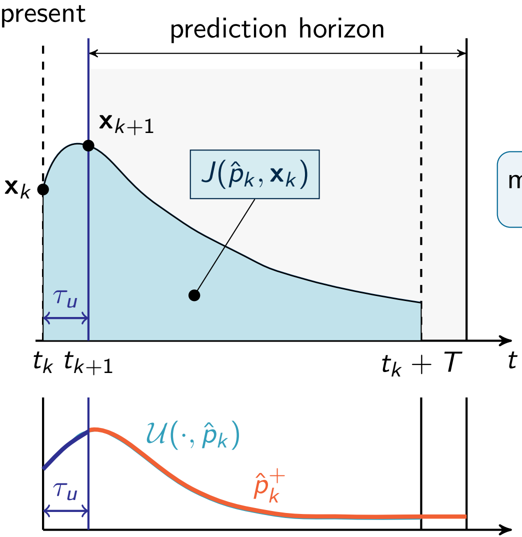

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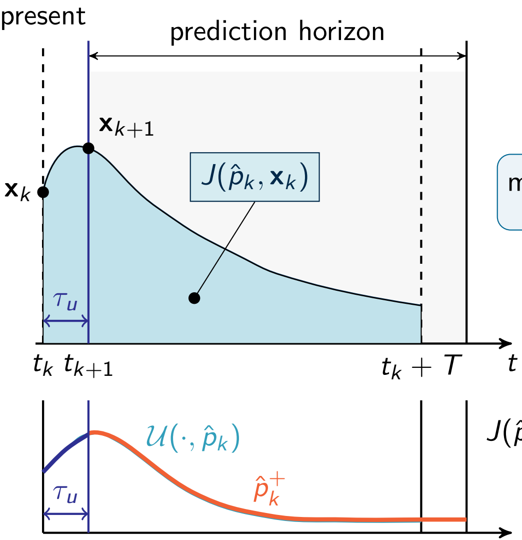

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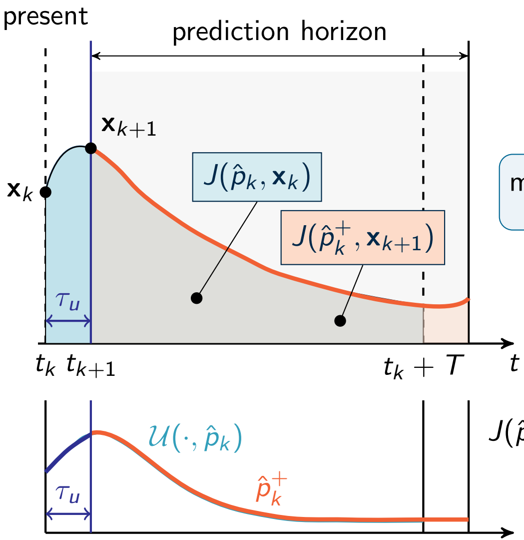

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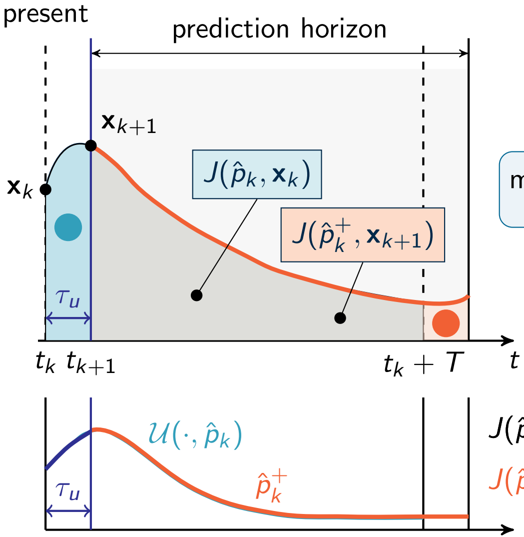

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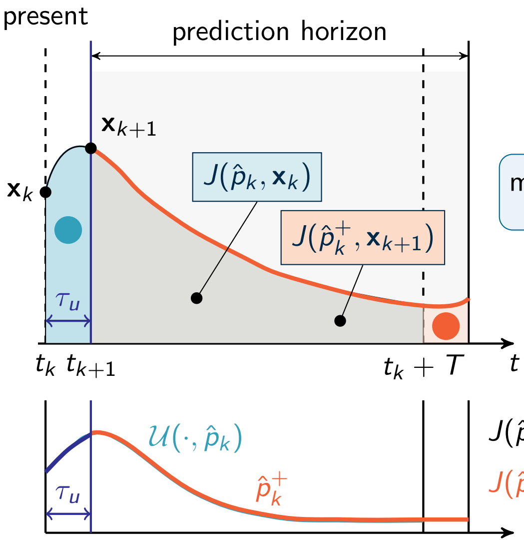

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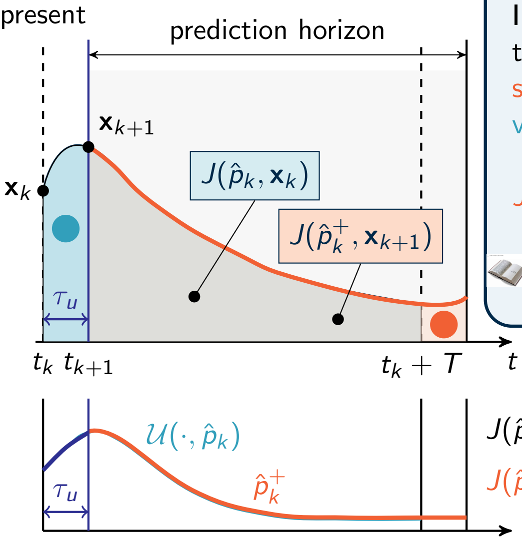

Keep in mind

In the **ideal framework**, when the horizon moves, the hot start $\hat{p}_k^+$ computed from the previous optimal solution $\hat{p}_k$ satisfies:

$$J(\hat{p}_k^+, \mathbf{x}_{k+1}) \leq J(\hat{p}_k, \mathbf{x}_k)$$

Mayne et al. Automatica (2000)

$$J(\hat{p}_{k+1}, \mathbf{x}_{k+1}) \leq J(\hat{p}_k^+, \mathbf{x}_{k+1})$$

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Ideal MPC: The key assumptions

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- (hot start)-compatible parametrization
- $\hat{p}_k$ sufficiently good

Ideal MPC: The key assumptions

Keep in mind

In a **realistic framework**, when the horizon moves, the hot start p_k^+ computed from the previous solution p_k satisfies:

$$J(p_k^+, \mathbf{x}_{k+1}) = J(p_k, \mathbf{x}_k) + D(\tau_u)$$

$D(0) = 0$
$D(\tau_u)$ is **not necessarily** ≤ 0.

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- (hot start)-compatible parametrization
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Ideal MPC: The key assumptions

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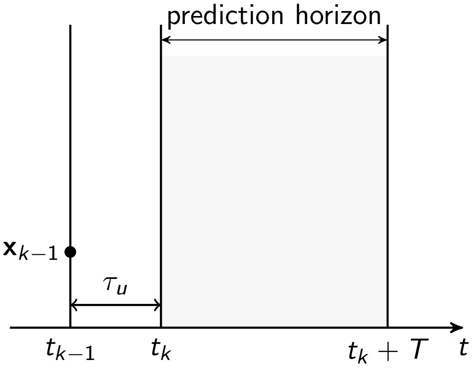
$D(\tau_u)$ is **not necessarily** ≤ 0.

Even with **perfect undisturbed** model

Preparation & Feedback Steps



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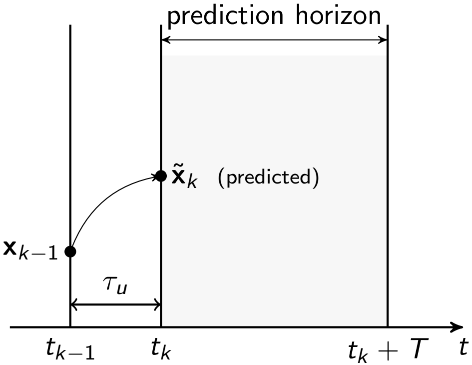

1. Predict $\tilde{\mathbf{x}}_k$
2. During $[t_{k-1}, t_k]$

   Compute $\hat{p}(\tilde{\mathbf{x}}_k)$ [and $\frac{\partial \hat{p}_k}{\partial \mathbf{x}}$]

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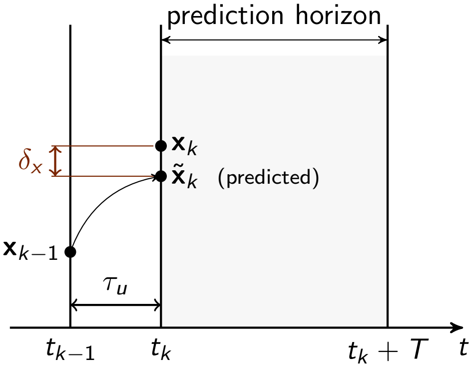

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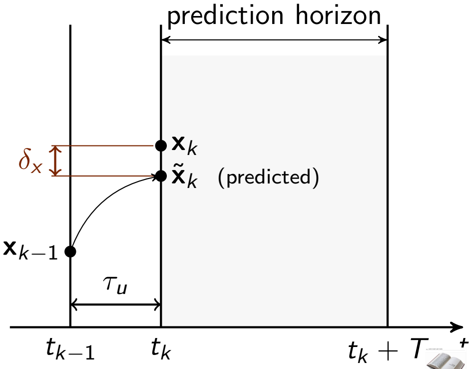

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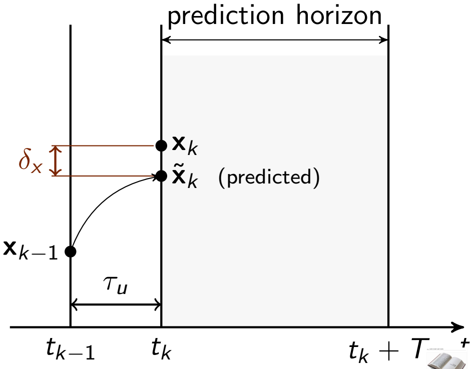
Diehl et al. SIAM J. Ctrl and Opt. (2005)

Zavala and Biegler. Auomatica (2009)

Preparation & Feedback Steps



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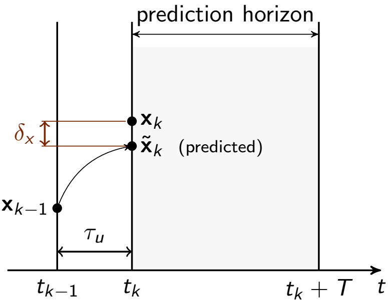
NOTA: Even with feedback, uncertainty potentially increases $D(\tau_u)$

Definition of Fast NMPC Problems



τ_u is the time between two control updating

τ_u is the time during which there is no feedback

$$\Rightarrow \tau_u \leq \tau_u^{max}$$

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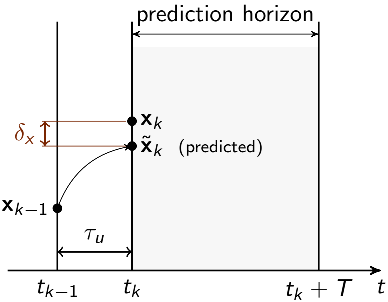

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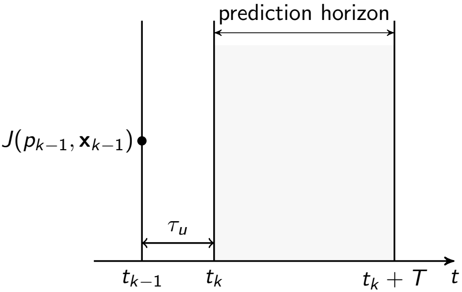
Fast NMPC Problems

Fast NMPC problems are those for which

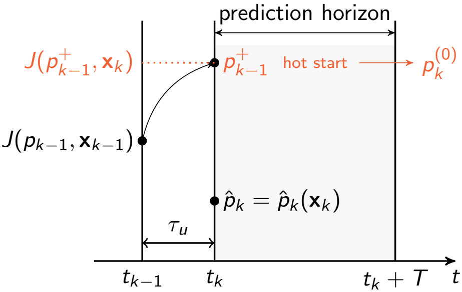
$$\tau_{solve}\Big(NLP(\tilde{\mathbf{x}}_k)\Big) \geq \tau_u^{max}$$

The Iterative Process



$$p^{(i+1)} \leftarrow \mathcal{S}(p^{(i)}, \mathbf{x})$$

The Iterative Process



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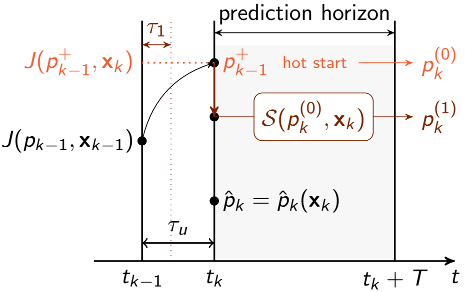

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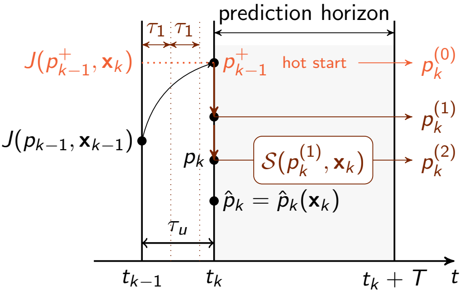

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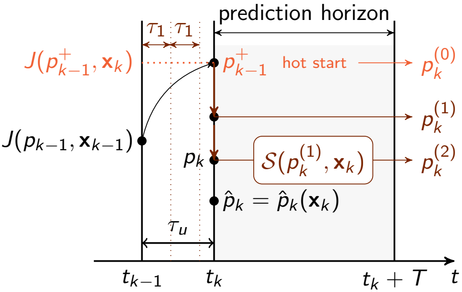
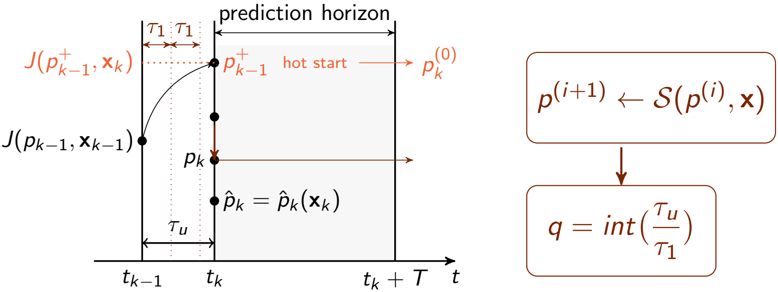


$$p^{(i+1)} \leftarrow \mathcal{S}(p^{(i)}, \mathbf{x})$$

$$q = int\left(\frac{\tau_u}{\tau_1}\right)$$

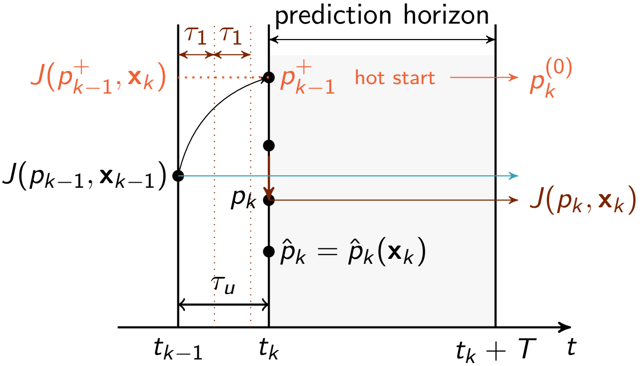
The Iterative Process



dynamic equation for p

$$p_k \quad := \quad \mathcal{S}^{(q)}(p^+_{k-1}, \mathbf{x}_{k-1})$$

Sufficient Conditions of Success



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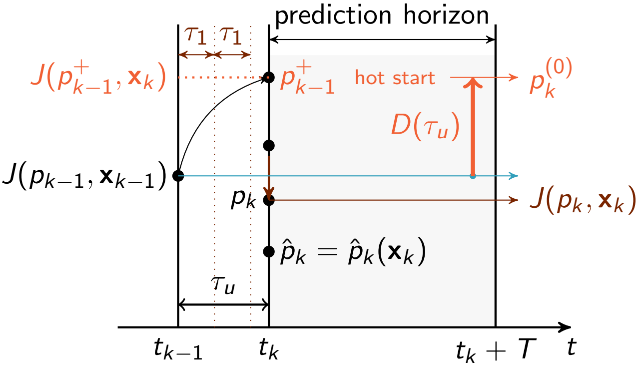

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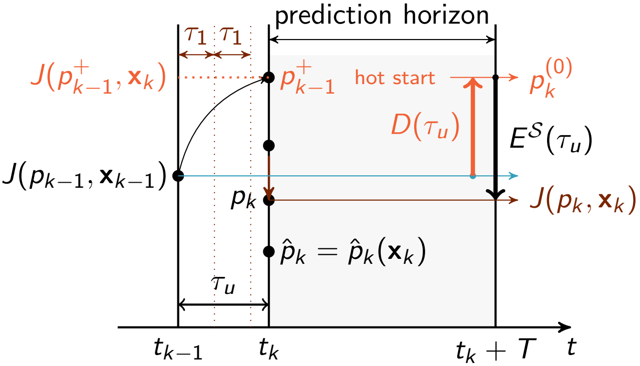
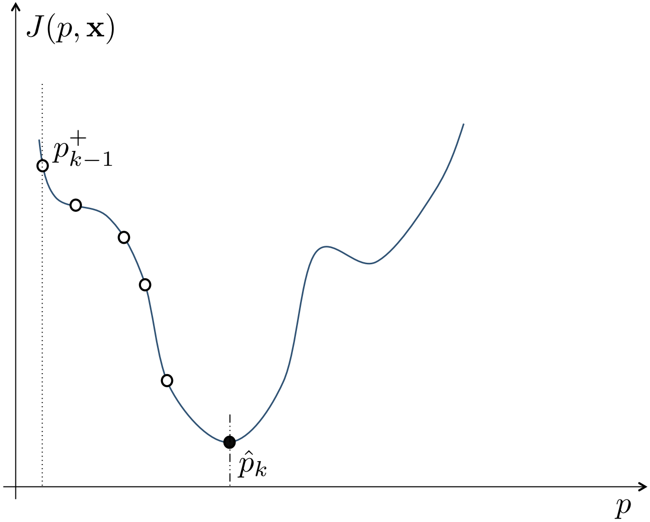


Closed-Loop Evolution of the Cost Function



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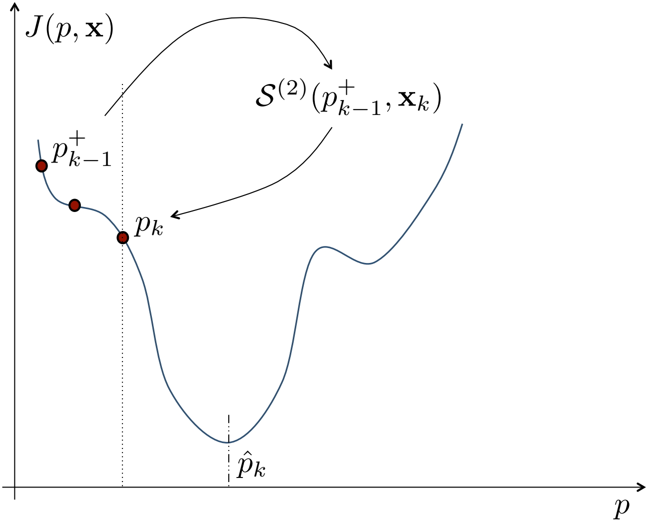

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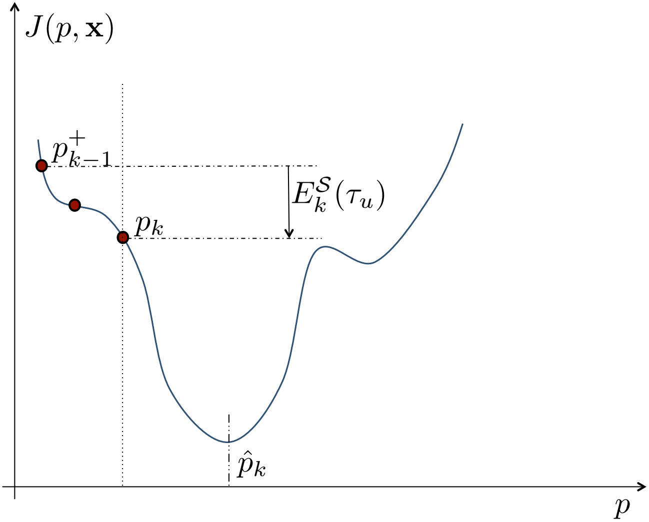

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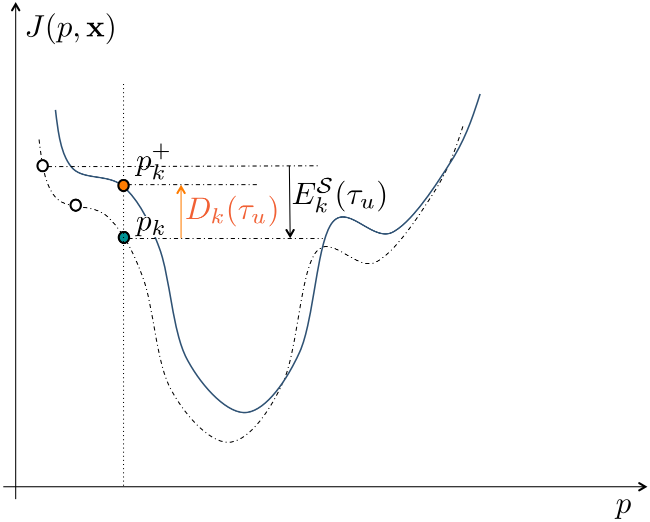

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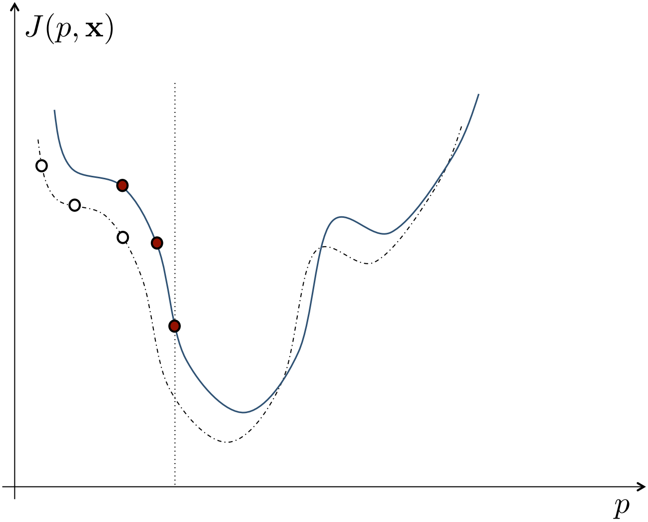

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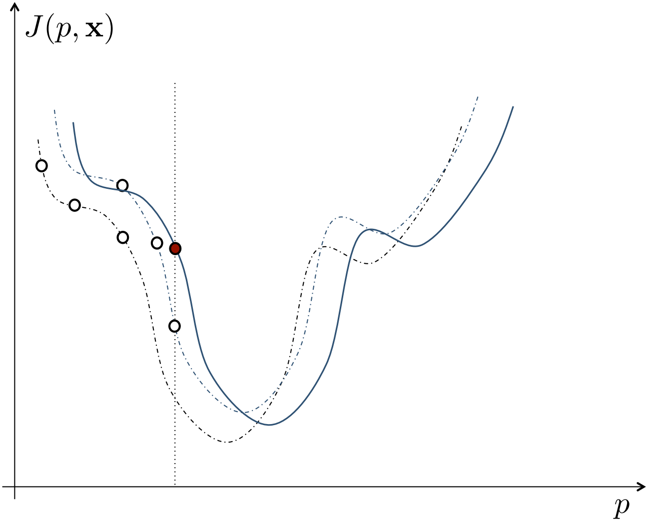

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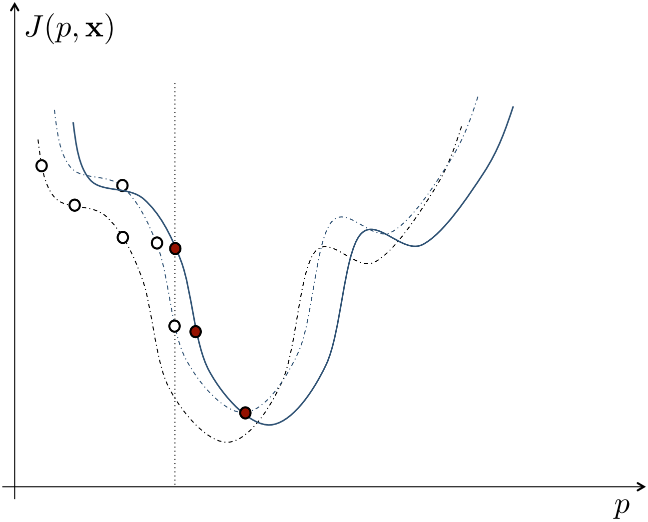

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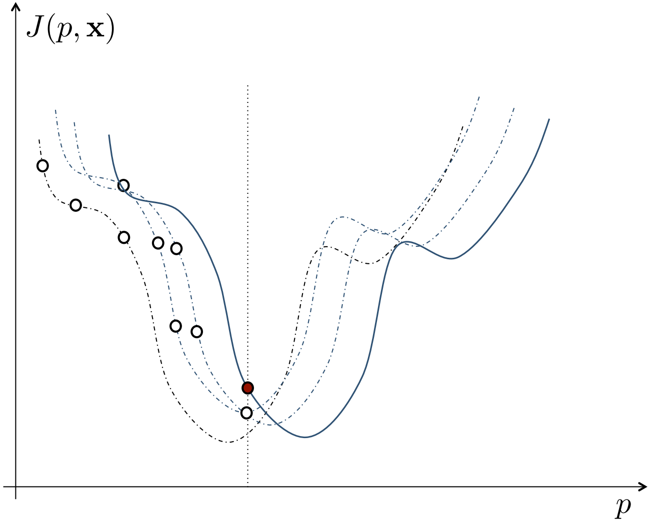

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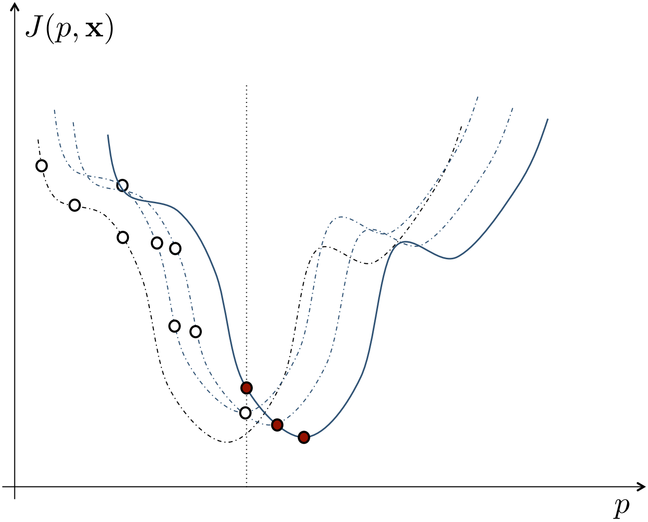

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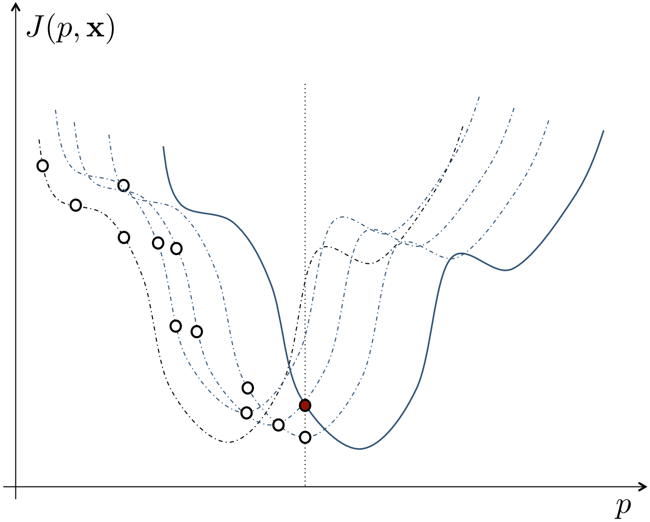

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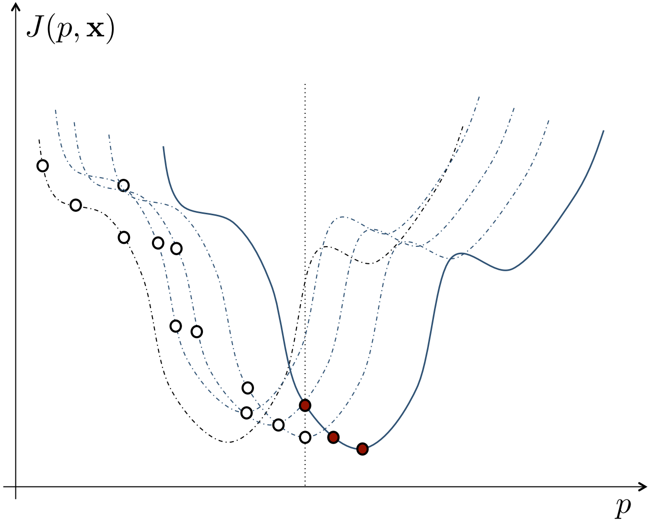

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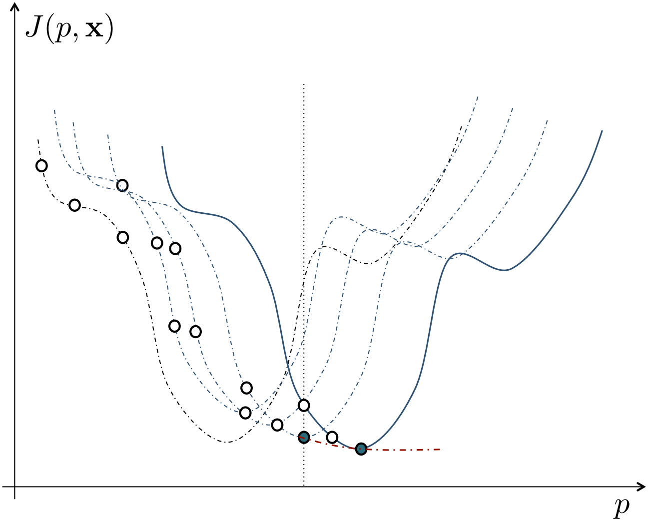

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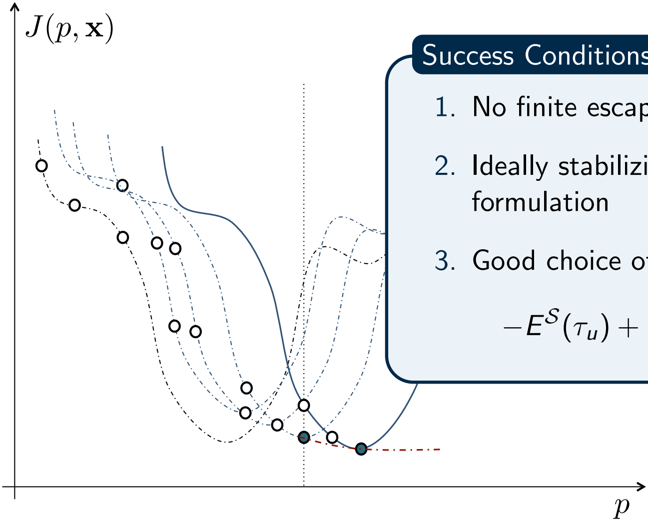
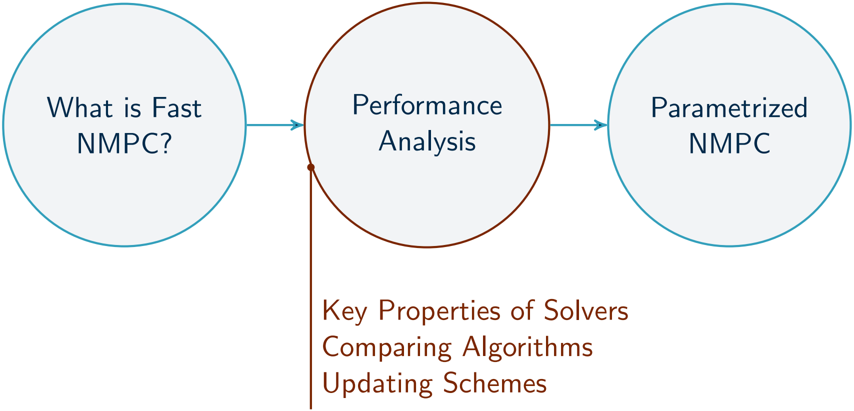


Success Conditions

1. No finite escape time
2. Ideally stabilizing NMPC formulation
3. Good choice of $(\mathcal{S}, \tau_u)$:

$$-E^{\mathcal{S}}(\tau_u) + D(\tau_u) < 0$$

Outline


What is Fast NMPC?
Performance Analysis
Parametrized NMPC
Key Properties of Solvers
Comparing Algorithms
Updating Schemes

Reminder

$$D(\tau_u) - E^{\mathcal{S}}(\tau_u) < 0$$

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$D(\tau_u)$

$-E^{\mathcal{S}}(\tau_u)$

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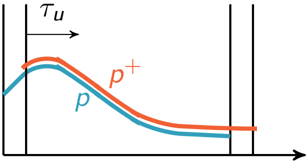
$D(\tau_u)$



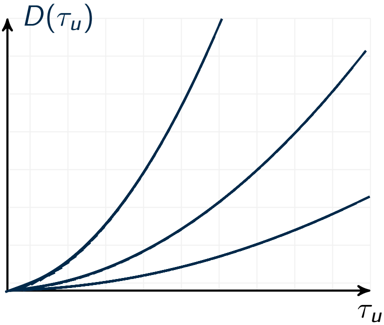
- $D(\tau_u) := J(p^+, \mathbf{x}^+) - J(p, \mathbf{x})$
- $D(0) = 0$, $D(\tau_u)$ can be ≥ 0
- $\tau_u \in [0, \tau_u^{max}]$
- Independent of the solver $\mathcal{S}$

$-E^{\mathcal{S}}(\tau_u)$

Reminder

$$D(\tau_u) - E^{\mathcal{S}}(\tau_u) < 0$$

$D(\tau_u)$



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$D(\tau_u)$

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- $E^{\mathcal{S}}(\tau_u) := J(p^{(0)}, \mathbf{x}) - J(p^{(q^{\mathcal{S}})}, \mathbf{x})$
- $q^{\mathcal{S}} = int(\tau_u/\tau_1^{\mathcal{S}})$
- $\tau_1^{\mathcal{S}}$ time for a single iteration
- $\tau_u \in \{1, 2, \dots\} \times \tau_1^{\mathcal{S}}$
- $E^{\mathcal{S}}(0) = 0$

Reminder

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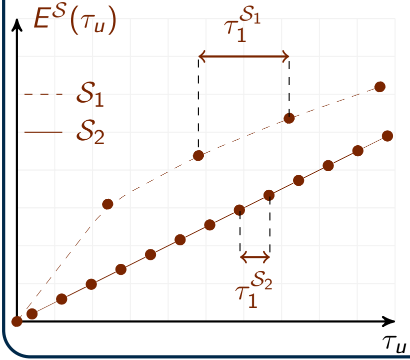
$D(\tau_u)$

$-E^{\mathcal{S}}(\tau_u)$



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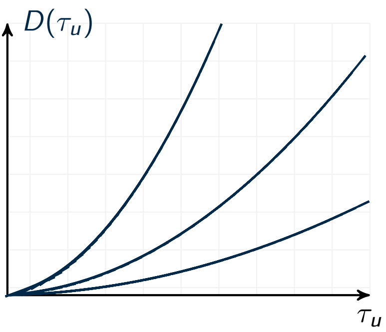
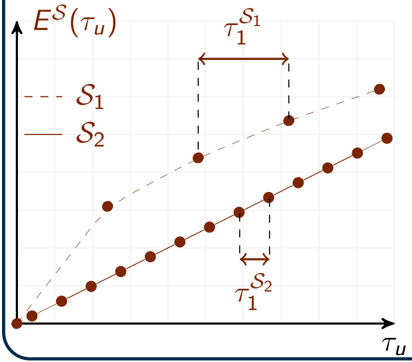
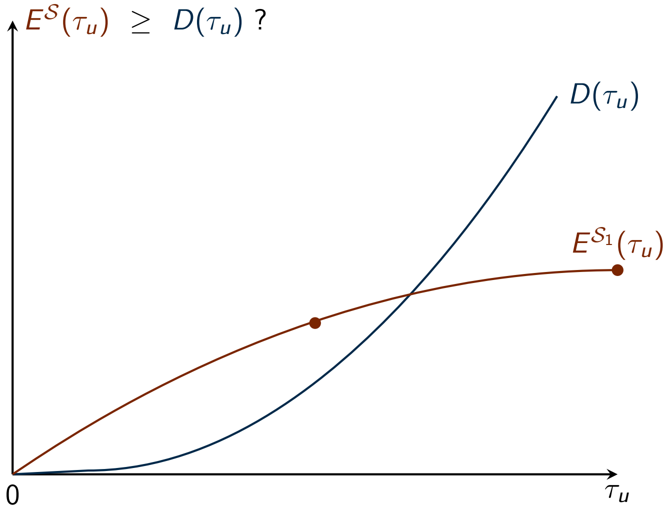
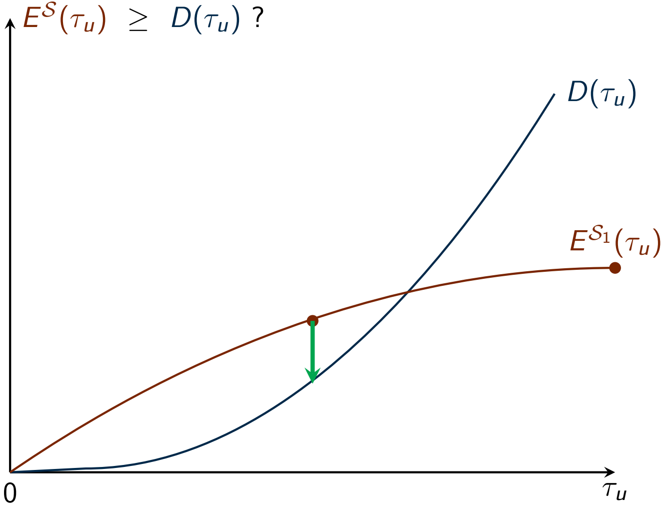


$-E^{\mathcal{S}}(\tau_u)$




$E^{\mathcal{S}}(\tau_u) \geq D(\tau_u)$?
$D(\tau_u)$
$E^{\mathcal{S}_1}(\tau_u)$
0
τ_u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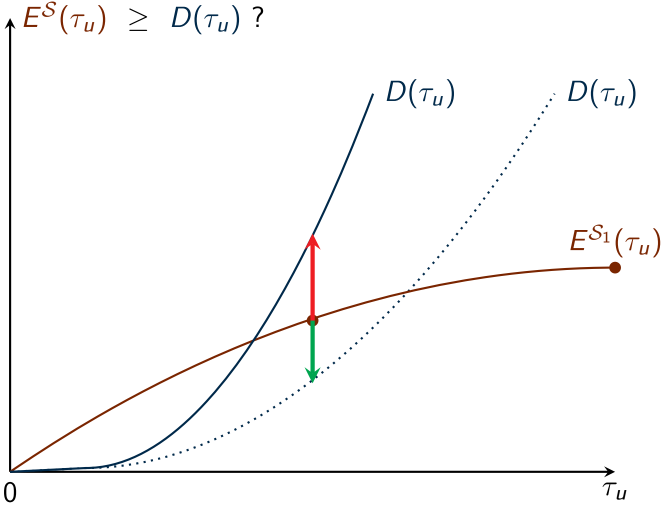

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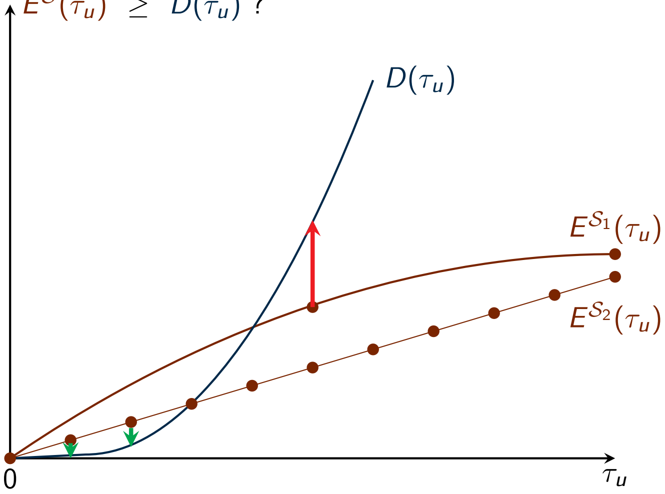
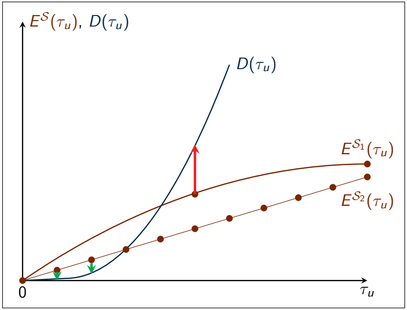
$E^{\mathcal{S}}(\tau_u) \geq D(\tau_u)$?



Key properties of a solver



Keep in Mind

It is sometimes better to choose a **less efficient**[a] solver with **shorter preparation step** duration τ_1.

[a] per iteration

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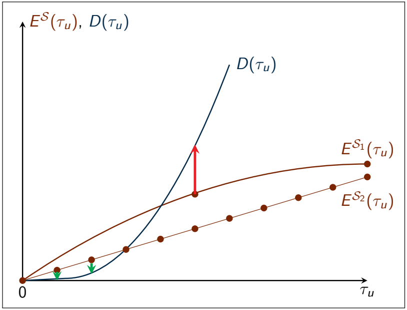

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It is sometimes better to choose a **less efficient**[a] solver with **shorter preparation step** duration τ_1.

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Gradient-based studies

Bemporad and Patrinos (2012), Jones et al. (2012), MA (2013).

Heuristics for second order methods

Bock et al. SIAM (2007)

Comparing Algorithms: An Example

Houska et al. Automatica (2011)

Model

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= u \qquad u \in [-1, +1] \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= -g \sin x_3 - u \cos x_3 - b x_4
\end{aligned}
$$

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Cost

$$
\sum_{i=1}^{N_p} \|\mathbf{x}_{k+i}^T\|_Q^2 + \|\mathbf{u}_{k+i-1}^T\|_R^2
$$

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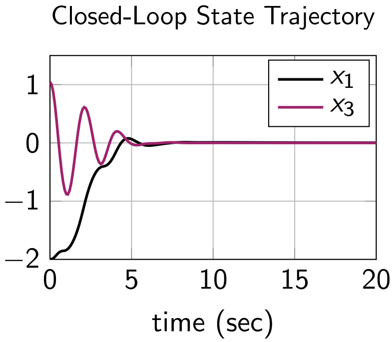
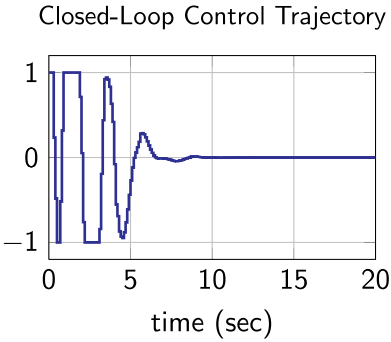
Cost

$$
\sum_{i=1}^{N_p} \|\mathbf{x}_{k+i}^T\|_Q^2 + \|\mathbf{u}_{k+i-1}^T\|_R^2
$$

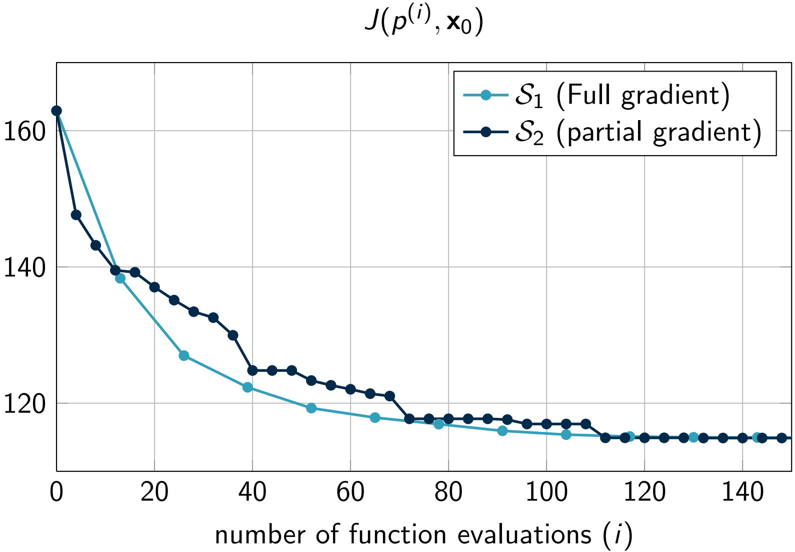
- $N_p = 10$ and $T_p = 3\ sec$
- Two solvers:
  - $\mathcal{S}_1$ Full gradient
  - $\mathcal{S}_2$ Partial gradient
- $\tau_1^{\mathcal{S}_2} = [\frac{4}{13}] \times \tau_1^{\mathcal{S}_1}$
- $\tau_u(\mathcal{S}_2) = [\frac{4}{13}] \times \tau_u(\mathcal{S}_1)$
- $\tau_u(\mathcal{S}_1) \in \{5, 50, 100\}\ ms$

Typical Closed-Loop Behavior

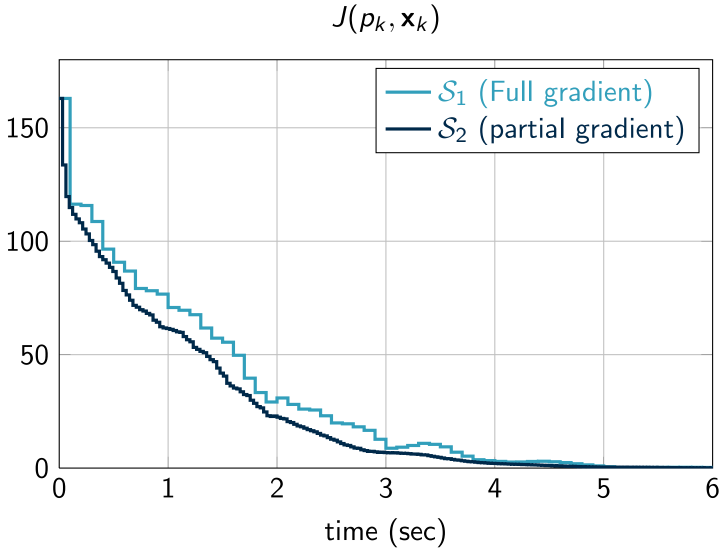




$\mathcal{S}_1$ is more efficient than $\mathcal{S}_2$ ... **per iteration** !



Closed-Loop Evolutions of the Predicted Cost



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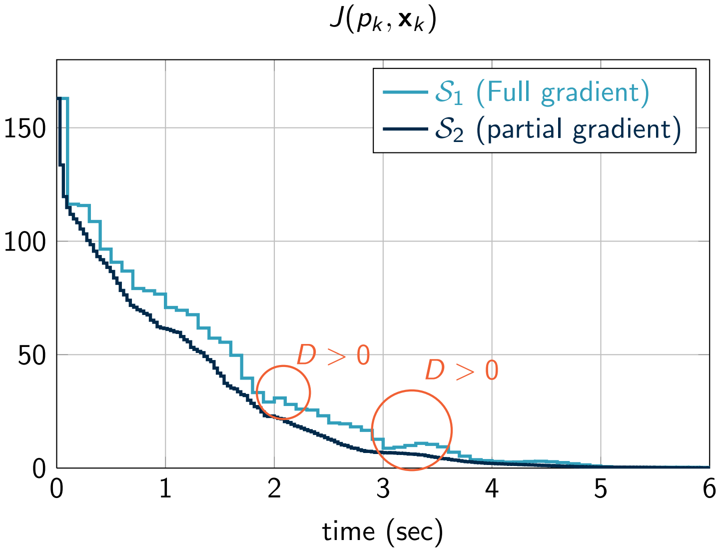


Comparison of Effective Closed-Loop Cost

$$J_{cl} := \frac{1}{N_{sim}} \sum_{k=1}^{N_{sim}} \left[\|\mathbf{x}_k\|_Q^2 + \|u_{k-1}\|_R^2 \right]$$

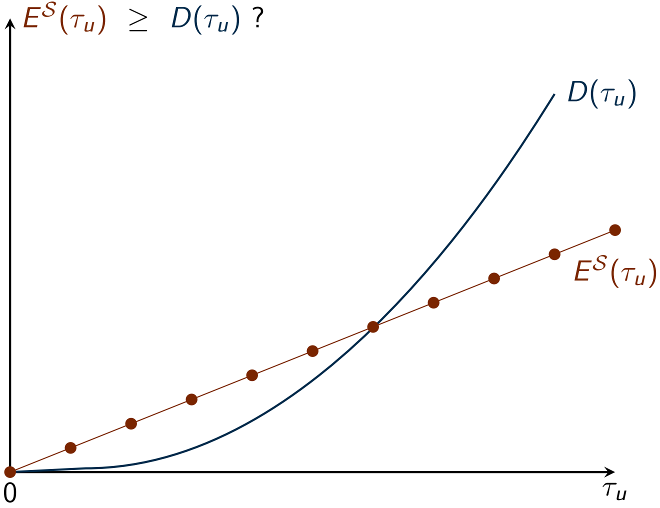
$\tau_u(\mathcal{S}_1)$	100 ms	50 ms	5 ms
Full Gradient	0.728	0.430	0.027
Partial Gradient	0.300	0.110	0.008
Gain %	59%	74%	70%

Updating Scheme For a Given Solver

Assume that a solver $\mathcal{S}$ has been chosen . . .

Is there any remaining choice ?

What is the optimal τ_u for a given solver?



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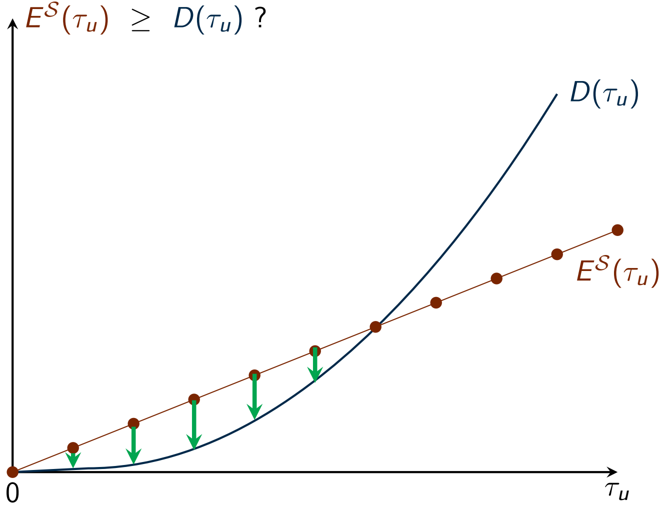

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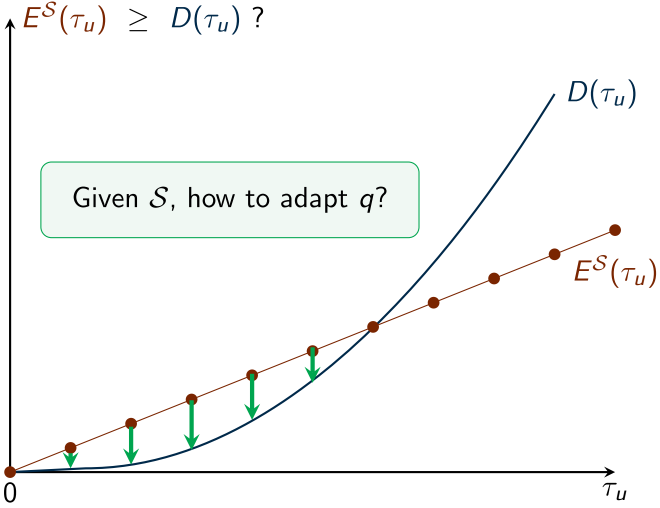


Updating τ_u is a control problem . . . !

$$p_{k+1} \quad = \quad \mathcal{S}^{(q(\tau_u))}\left(p_k{}^{+}, \mathbf{x}_k\right)\Big)$$

Updating τ_u is a control problem ...!

$$
\begin{aligned}
p_{k+1} &= \mathcal{S}^{(q(\tau_u))}\big(p_k{}^+, \mathbf{x}_k\big) \\
\mathbf{x}_{k+1} &= f\big(\mathbf{x}_k, \mathcal{U}(0, p_k)\big)
\end{aligned}
$$

Updating τ_u is a control problem . . . !

$$\begin{pmatrix} p_{k+1} \\ \mathbf{x}_{k+1} \end{pmatrix} = F\left(\begin{pmatrix} p_k \\ \mathbf{x}_k \end{pmatrix}, \tau_u\right)$$

Updating τ_u is a control problem ...!

$$\begin{pmatrix} p_{k+1} \\ \mathbf{x}_{k+1} \end{pmatrix} = F\left(\begin{pmatrix} p_k \\ \mathbf{x}_k \end{pmatrix}, \tau_u\right)$$

$$y = J(p_k, \mathbf{x}_k)$$

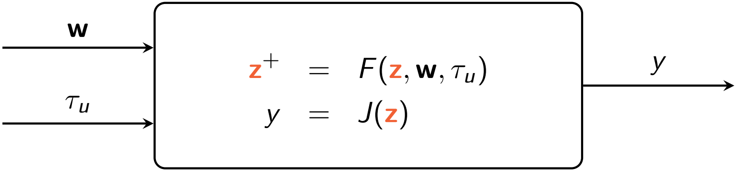
Updating τ_u is a control problem ...!

$$
\begin{aligned}
\mathbf{z}^{+} &= F(\mathbf{z}, \tau_u) \\
y &= J(\mathbf{z})
\end{aligned}
$$

Updating τ_u is a control problem . . . !

$$\begin{aligned} \mathbf{z}^+ &= F(\mathbf{z}, \mathbf{w}, \tau_u) \\ y &= J(\mathbf{z}) \end{aligned}$$

Updating τ_u is a control problem . . . !



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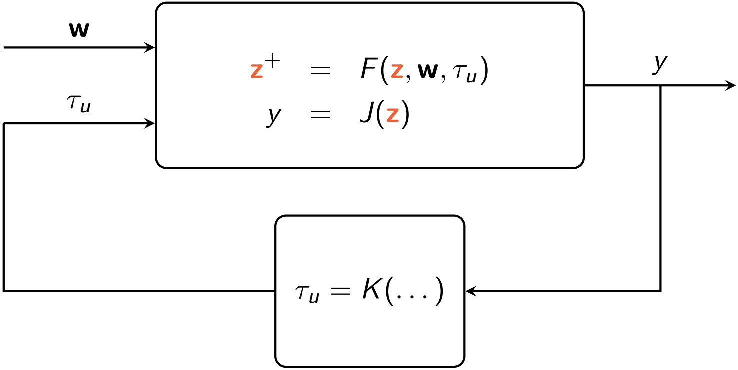

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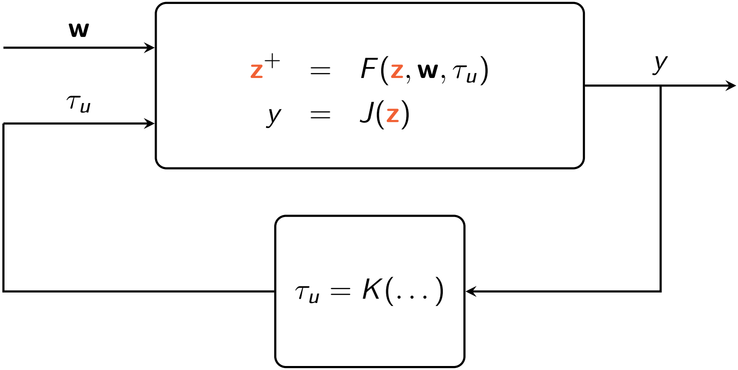
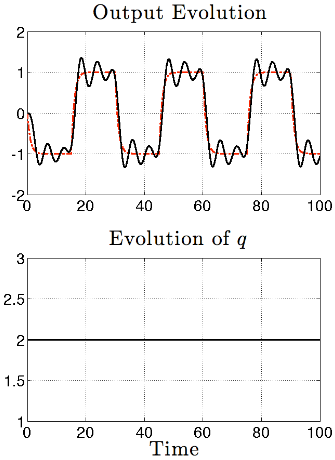


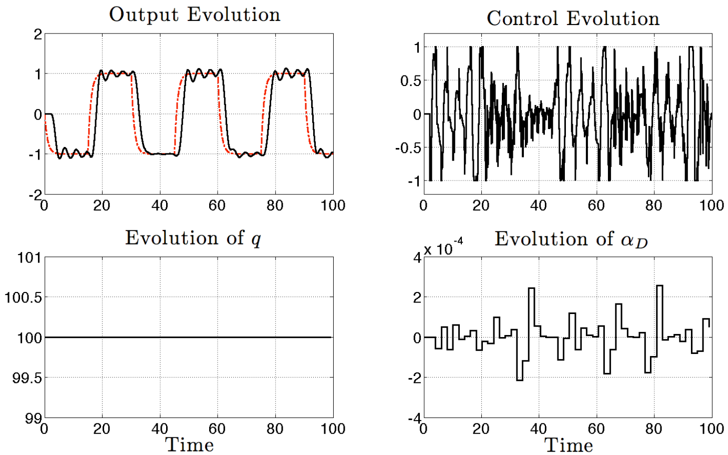
M.A. ECC (2013)

$q = 2$ without adaptation



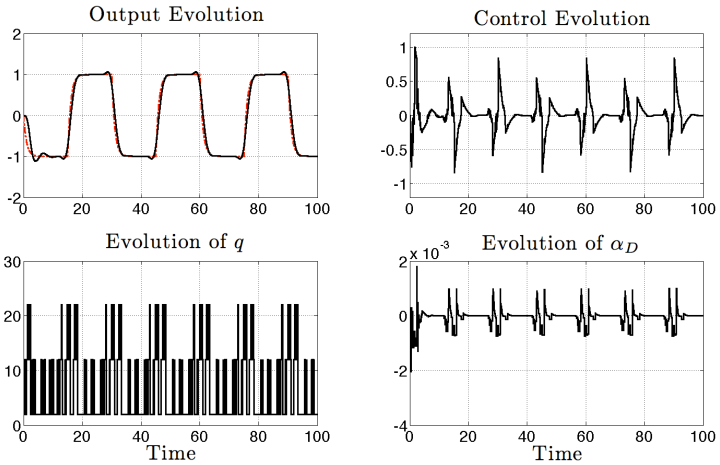
MA, ECC (2013)

$q = 100$ without adaptation



MA, ECC (2013)

$q^{(0)} = 2$ with adaptation



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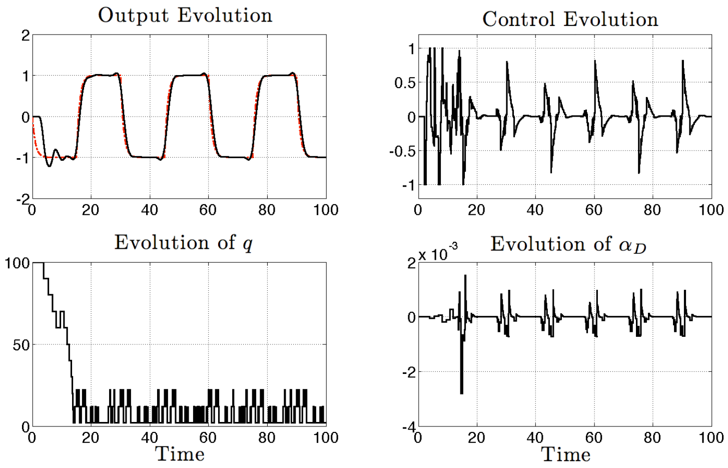
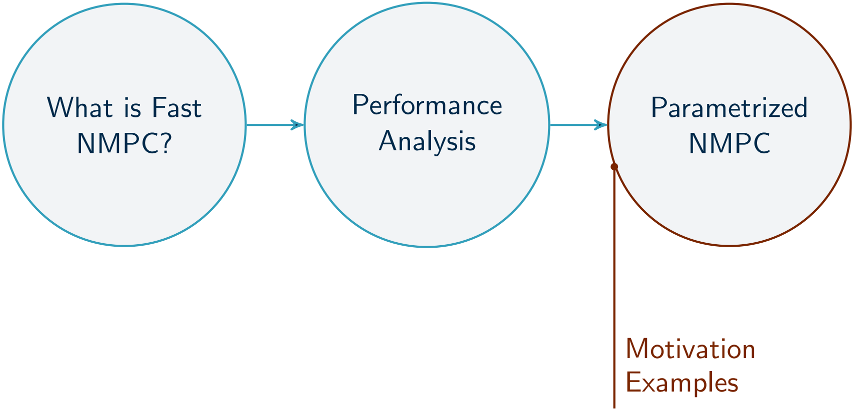
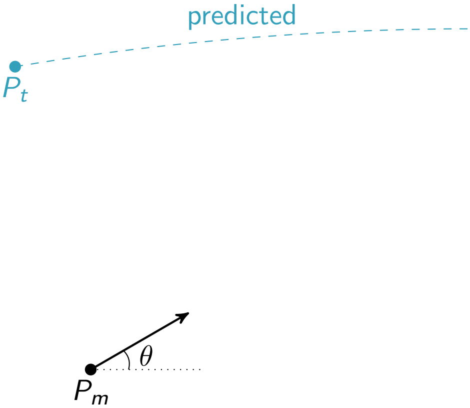


MA, ECC (2013)

Outline


What is Fast NMPC?
Performance Analysis
Parametrized NMPC
Motivation
Examples

Introductory Example: Interception of moving target



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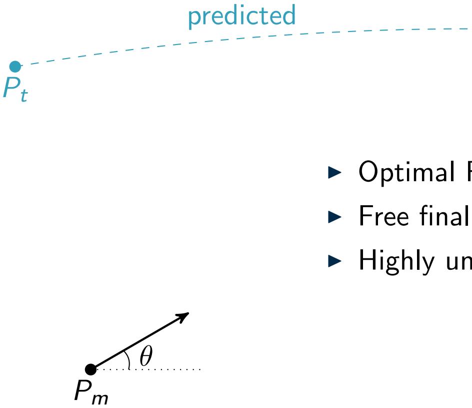
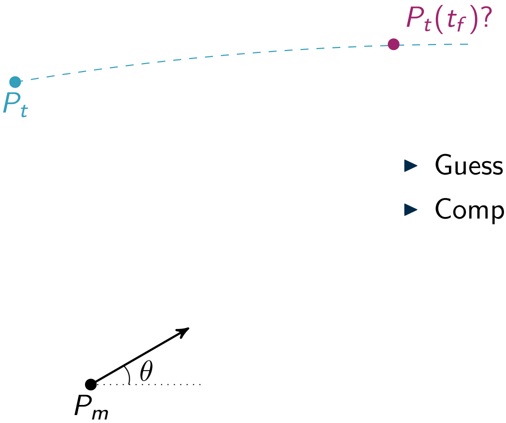


- Optimal Rendez-vous problem
- Free final time
- Highly uncertain

Introductory Example: Interception of moving target



- Guess some t_f.
- Compute the corresponding $P_t(t_f)$

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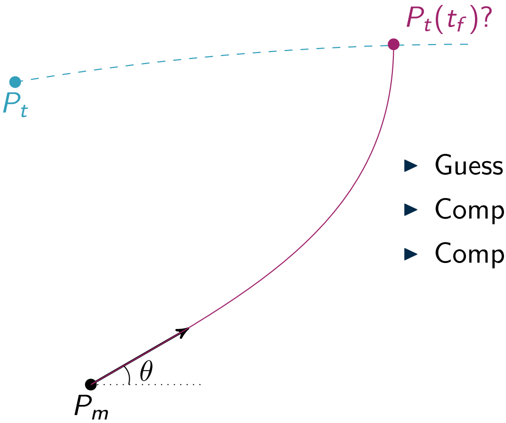

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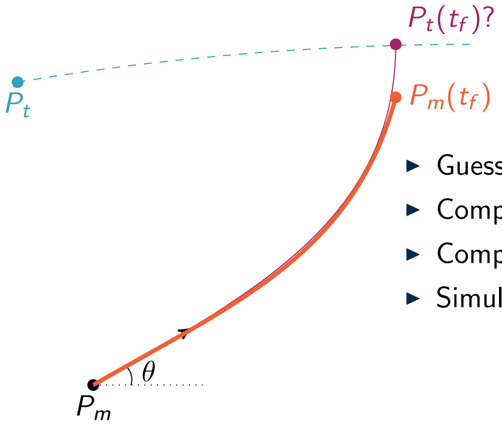

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- Simulate parabola tracking

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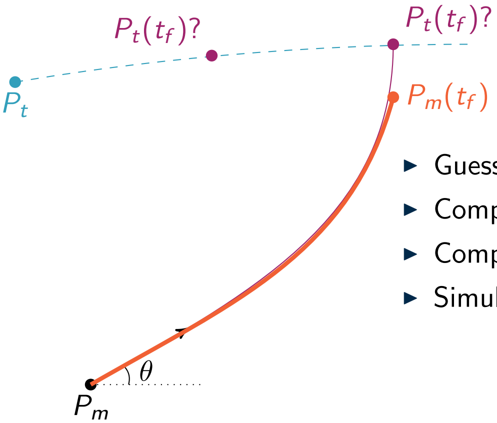

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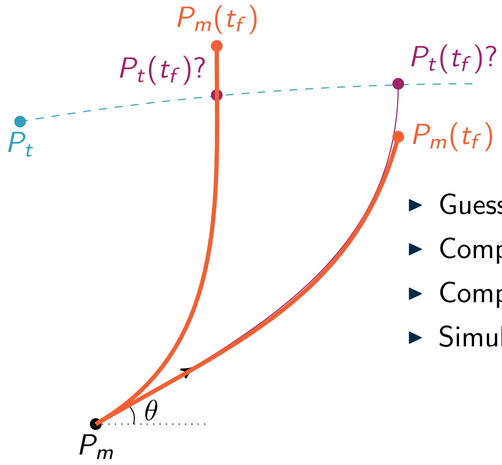

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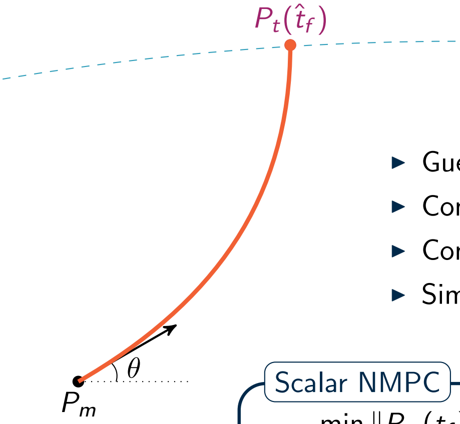

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Scalar NMPC

$$\min_{t_f} \|P_m(t_f) - P_t(t_f)\|$$

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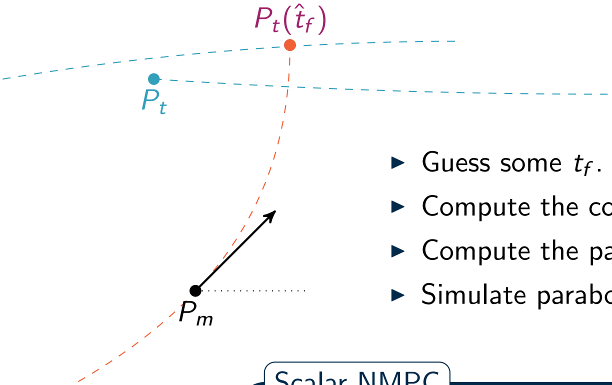

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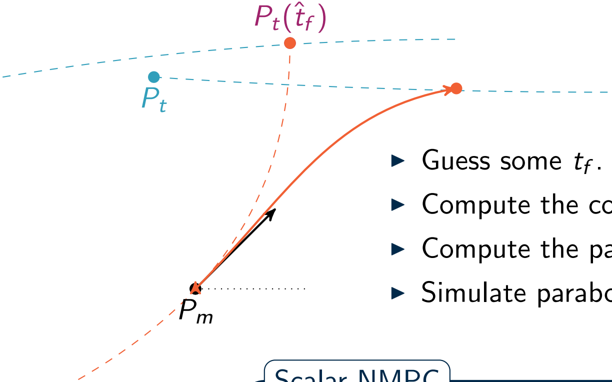

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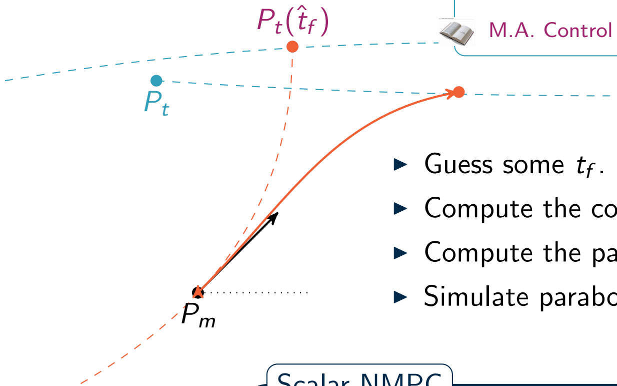
Introductory Example: Interception of moving target

Full solution	40	ms
Fast solution	0.4	ms

M.A. Control Eng. Practice (2001)



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General Aspects of Parametrized NMPC

- Reduced number of parameters
- Problem conditioning
- Sub-optimality
- Problem dependent

1. Reduce τ_1 !
2. Qualify specific algorithms (Simplex, Torczon, etc.)
3. Easier certification

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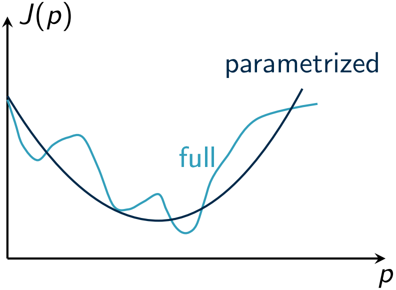

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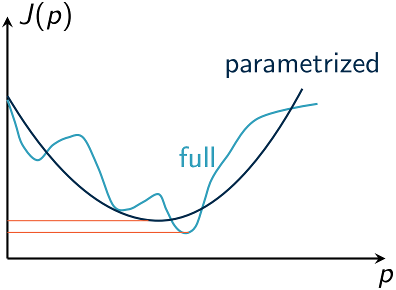

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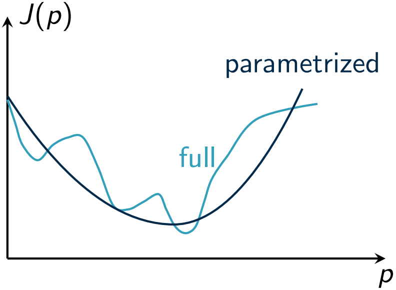

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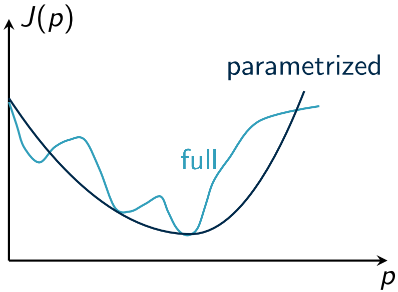

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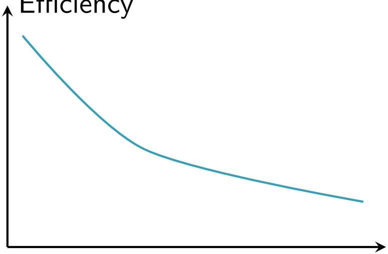
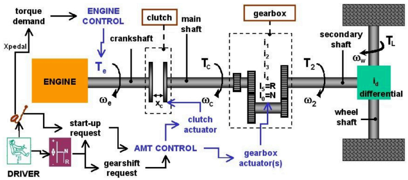


Automated Manual Transmission



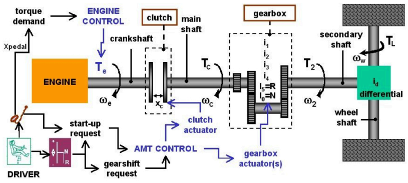
R. Amari

P. Tona (IFP)

Amari et al. IFAC World Congress (2008)

Automated Manual Transmission


R. Amari | P. Tona (IFP)



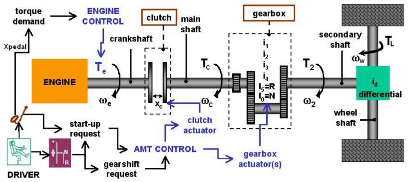
Control objective

- Smooth ($\omega_e - \omega_c \rightarrow 0$)
- Transparency (pedal $\rightarrow T_e$)

Amari et al. IFAC World Congress (2008)

Automated Manual Transmission

R. Amari P. Tona (IFP)



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Constraints

- Control level saturation
- Control derivative saturation
- Embedded control

Amari et al. IFAC World Congress (2008)

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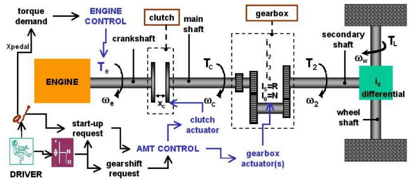

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Solution

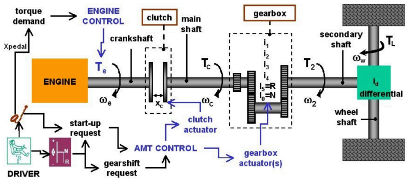
Replace NL groups by observed terms.

Parametrized MPC with $p = T_p$

Solve the NLP by dichotomy

Amari et al. IFAC World Congress (2008)

Automated Manual Transmission



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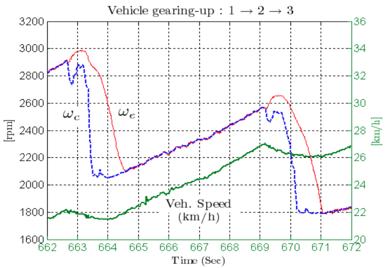

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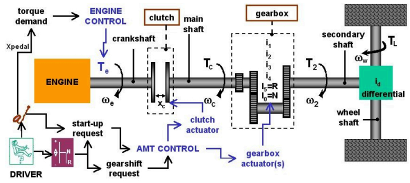


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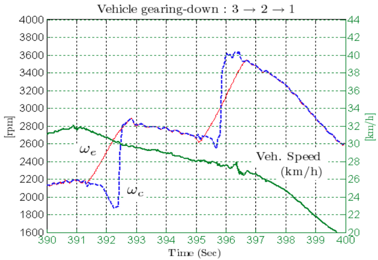


Control objective

- Smooth ($\omega_e - \omega_c \to 0$)
- Transparency (pedal $\to T_e$)

Constraints

- Control level saturation
- Control derivative saturation
- Embedded control

Solution

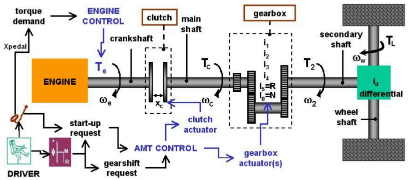
Replace NL groups by observed terms.

Parametrized MPC with $p = T_p$

Solve the NLP by dichotomy

Amari et al. IFAC World Congress (2008)

Automated Manual Transmission



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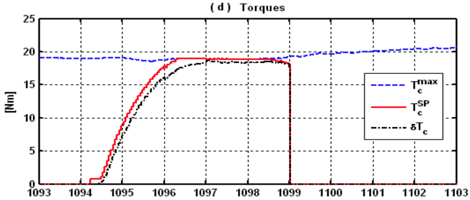

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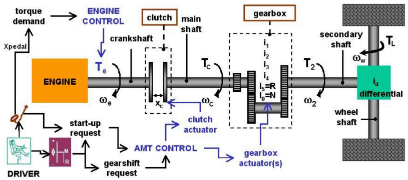

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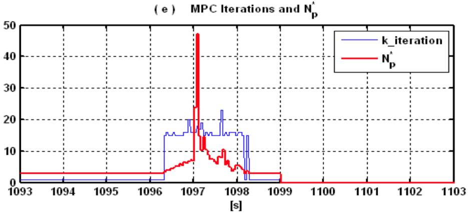

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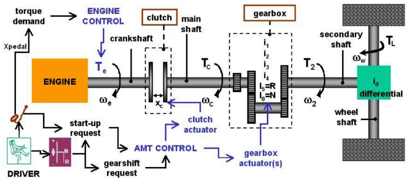


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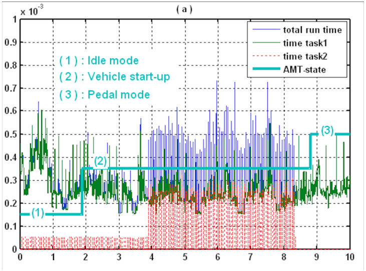

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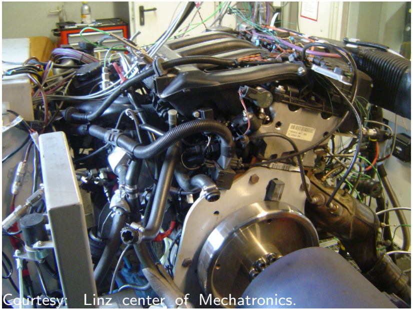
Diesel Engine AirPath Control



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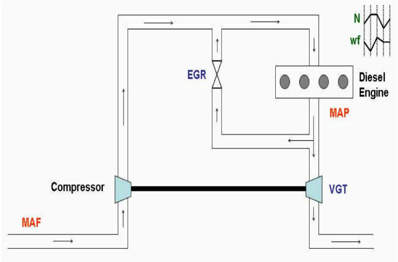

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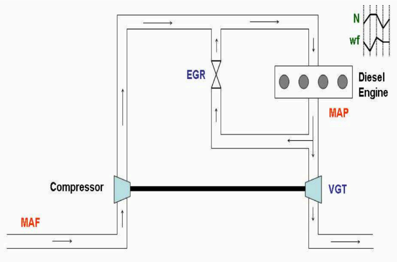


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Problem description

Nonlinear 8-dimensional model

Non minimum-phase dynamic

Saturations on u and $\dot{u}$

Embedded Control

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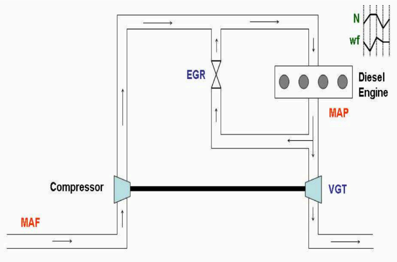

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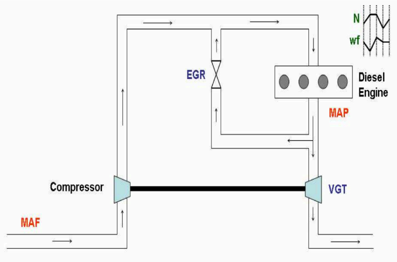


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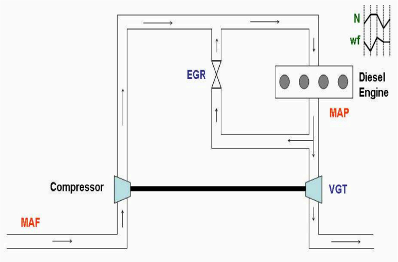

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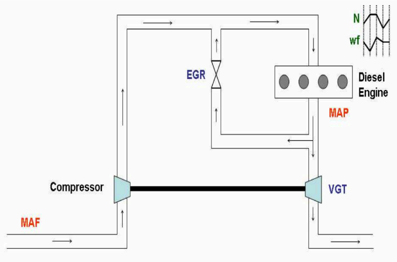

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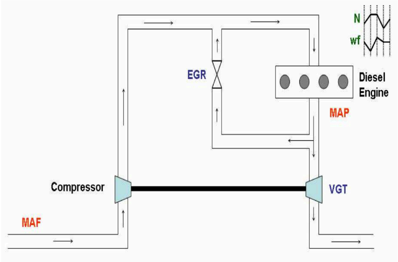

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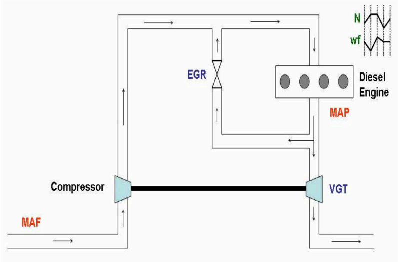



A. Murilo P. Ortner (Linz) R. Furhapter (Linz)

Parameters of the solution

Model sampling period	50 ms
Prediction horizon	$30 \times \tau$
SQP / Trust region	
Number of function eval.	30

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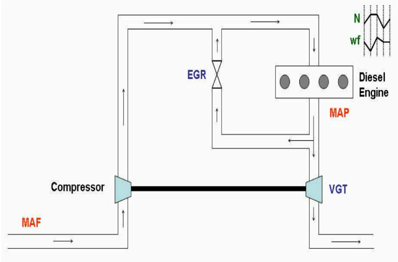

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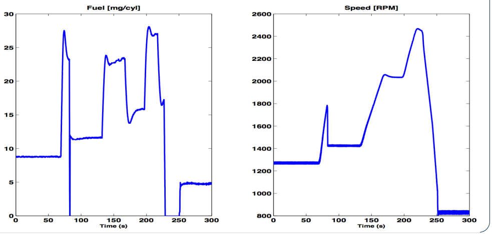

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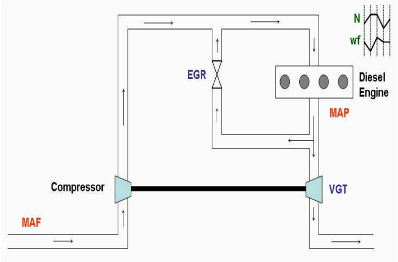

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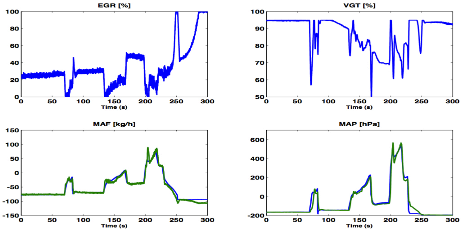

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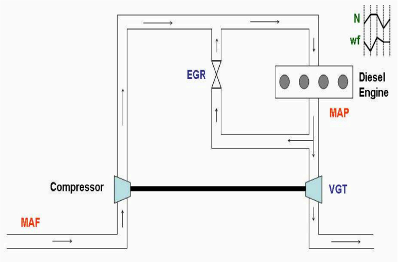

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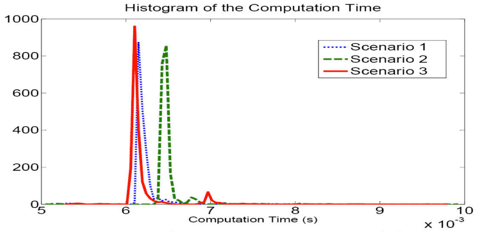

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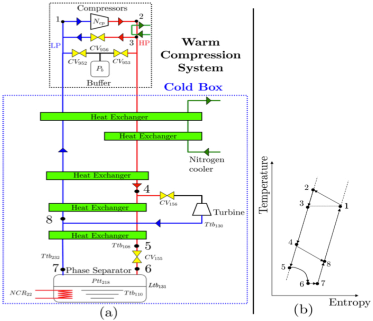
Control of a Cryogenic System



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Bonne et al. J. Process Control (2014)

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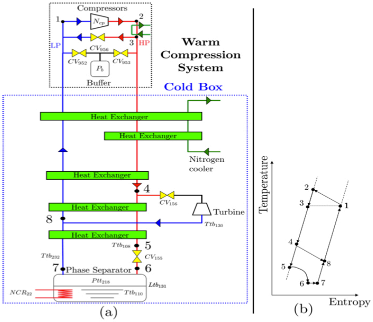

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Despite the context, hard limitations:

- Computation power
- Memory use

(Schneider TSX Premium automata)

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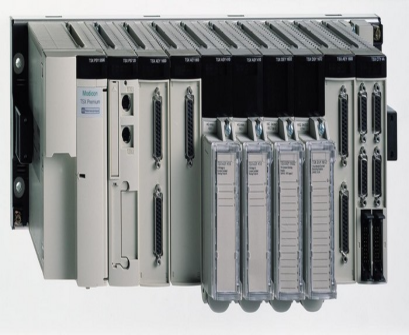

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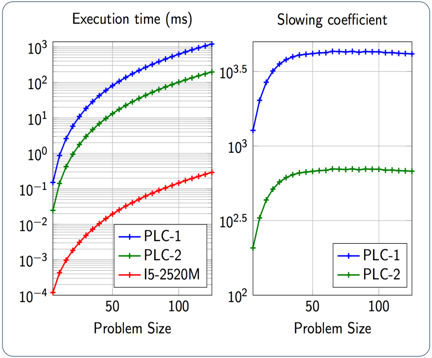

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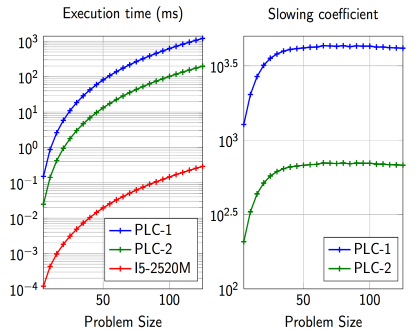

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⇒ This was a fast NMPC problem !!

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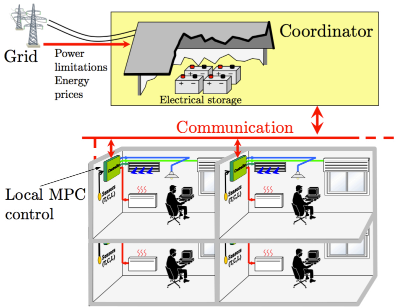

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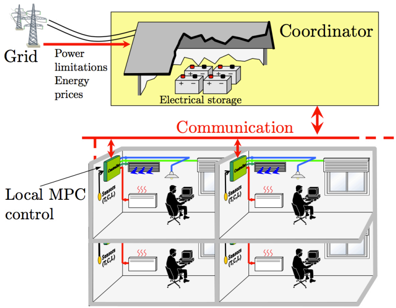

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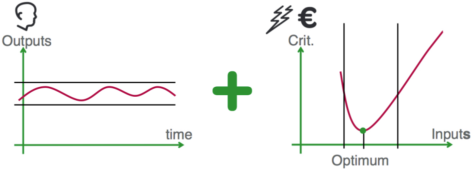

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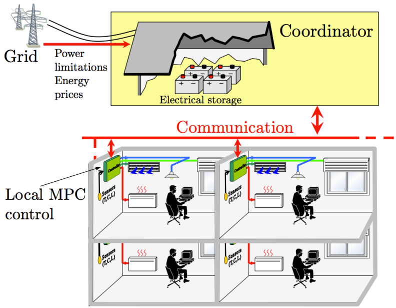

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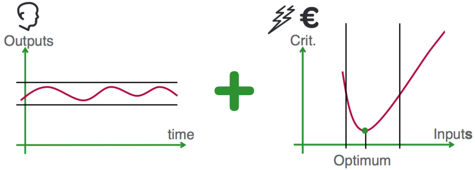

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- Solve using fixed-point iterations.

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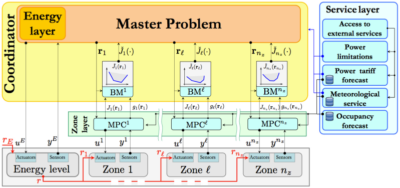

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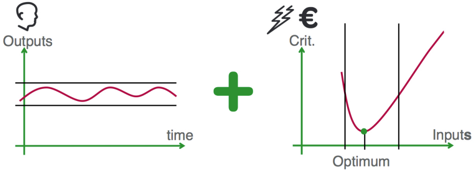
Need fast local solution in order to

1. Enable Master iterations
2. Perform yearly simulations



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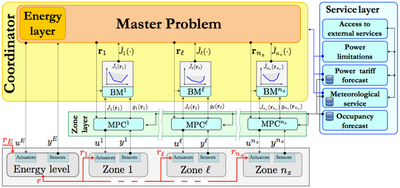

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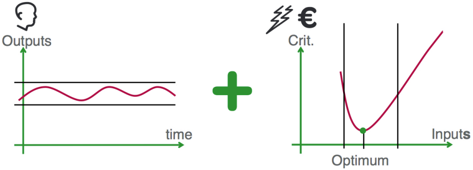

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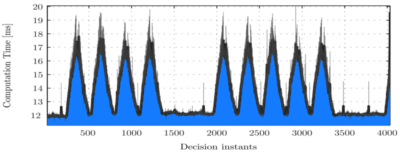

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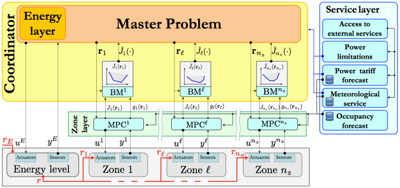

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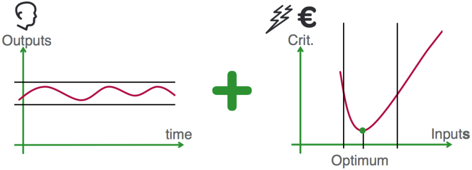

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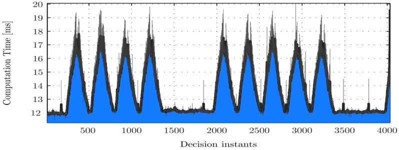

Data for 20 zones, $\tau_u = 5$ min

- 2×10^6 optimization pb
- 900 dof, 1000 constraints
- Simulation time = 18 h.

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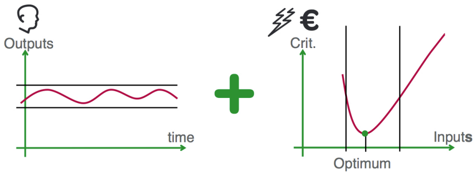

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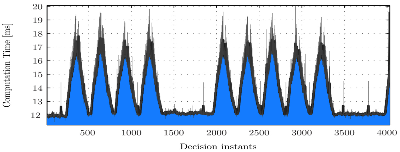

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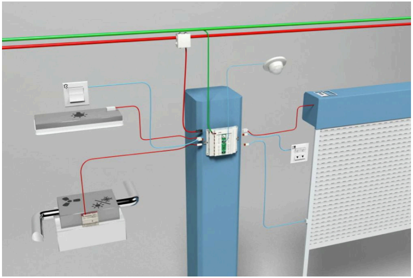
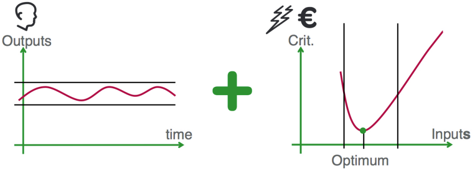
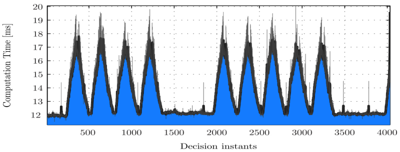


M. Y. Lamoudi (Schneider Electric Industry)



- Constrained nonlinear NLP.
- Solve using fixed-point iterations.



Data for 20 zones, $\tau_u = 5$ min

- 2×10^6 optimization pb
- 900 dof, 1000 constraints
- Simulation time = 18 h.

Lamoudi et al. in *distributed MPC made easy* Springer (2014)

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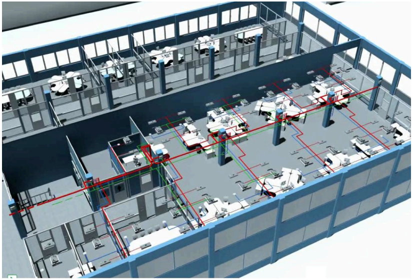

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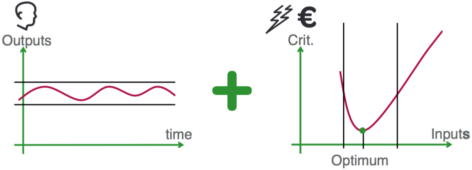

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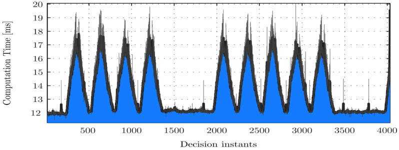

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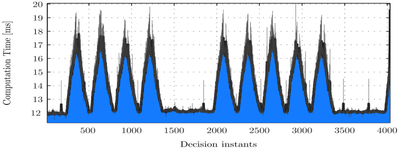
M. Y. Lamoudi (Schneider Electric Industry)

Room-Box data

- Prediction horizon = 12h
- 8.2% memory usage
- 6 sec to solve the NLP.



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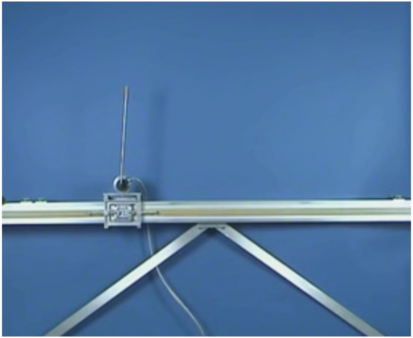
Swing-up of a Twin Pendulum System


A. Murilo


G. Buche



- scalar NMPC
- 5 system integration / iteration
- control updating period 20 ms.

M.A. and A. Murilo, Automatica (2008)

Conclusion

Good News

- Huge progress
- Free dedicated software
- MPC-reflex in expansion
- Promising future

Keep in Mind

- MPC needed for transient
- Our proofs are tautologies
- Heavy use of heuristics
- Keep feet on earth